On Economic Optimality of Model Predictive Control

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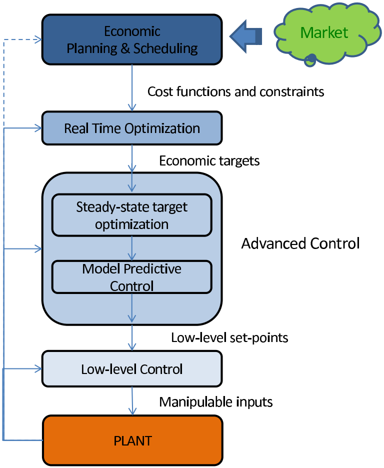
Outline

Real Time Optimization

Hierarchical control structure (Qin & Badgwell (2003); Engell (2007)):



1. The Real Time Optimizer (RTO) computes the operation point according to economic criteria and operation limits.
2. The RTO solves an optimization problem based on a complex nonlinear stationary model of the plant.
3. The setpoints computed by the RTO are sent to the MPC control.
4. MPC solves a QP based on a simplified dynamic model of the plant and constraints.

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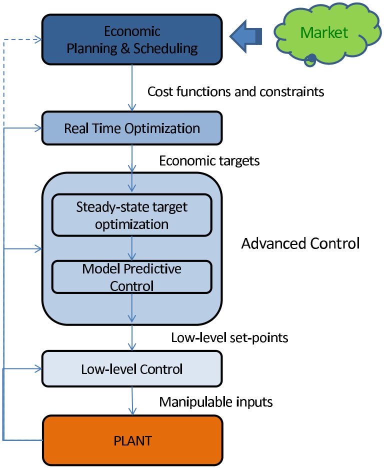

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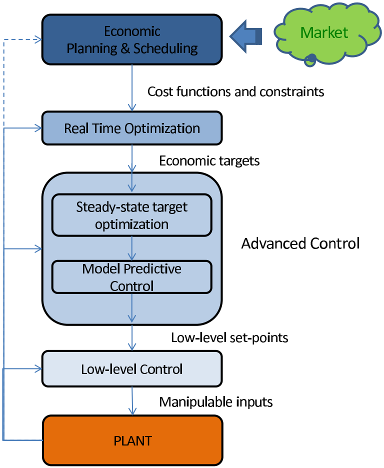

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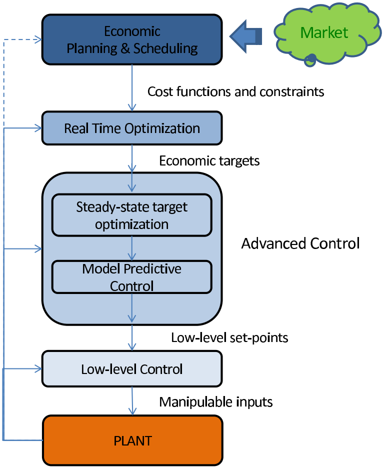

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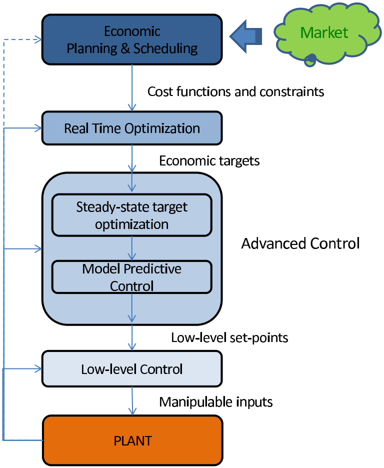

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Drawbacks of the control structure

- RTO time scale (hours or days) VS MPC time scale (minutes).
- Slow reaction to process variations (disturbances).
- There exist mismatches between the models of RTO and MPC.
- The RTO may provide inconsistent setpoints to the MPC.
  - Unreachability of the setpoints.
  - Poor economic performances.

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Existing solutions

- SSTO or LP/QP-MPC (Muske (1997); Ying & Joseph (1999); Marchetti et al. (2014))
  - Same model as the MPC.
  - Same sample time as the MPC.

- One-layer solutions
  - Dynamic RTO (Biegler (2009); Würth et al. (2009))
  - One layer RTO+MPC (Adetola & Guay (2010); Zanin et al. (2002))

- Economic MPC (Amrit (2011); Angeli et al. (2012); Diehl et al. (2011); Ferramosca et al. (2014))

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Economic Optimality - Comparative Study

Strategies taken into account

- MPC for tracking (Limon et al., 2008; Ferramosca et al., 2009).
- One-layer RTO+MPC (Zanin et al., 2002).
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Criteria:

- Fulfillment of economic objectives.
- Computational burden.
- Stability.

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Problem Statement

- Consider a system described by a discrete-time linear time-invariant model

$$\begin{aligned} x^+ &= Ax + Bu \\ y &= Cx \end{aligned}$$

- The system is subject to hard constraints on state and input:

$$x \in X, \quad u \in U$$

where $X \subset \mathbb{R}^n$ and $U \subset \mathbb{R}^m$ are compact sets.

Assumption

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RTO Problem

- The economic criterion is given by the function

$$f_{eco}(x, u, p)$$

where p is a vector of parameters which takes into account prices, costs, production goals, etc.

- The economic criterion to be optimized may change according to the market, the plant scheduling, or the data reconciliation tasks.
- The optimal steady state provided by the RTO is:

$$\begin{aligned} (x_s, u_s, y_s) \quad = \quad & \arg\min_{x,u,y} f_{eco}(x, u, p) \\ s.t. \quad & x = Ax + Bu,\ y = Cx, \\ & x \in X, \quad u \in U \end{aligned}$$

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MPC for tracking: MPCT

- Cost function designed as a measure of the distance of the predicted trajectory to the economic setpoint y_s.

$$V_N^t(x, y_s; \mathbf{u}) = \sum_{j=0}^{N-1} \|x(j) - x(N-1)\|_Q^2 + \|u(j) - u(N-1)\|_R^2 + V_O(y(N-1), y_s)$$

where $x(j) = \phi(j; x, \mathbf{u})$, $Q > 0$, and $R > 0$.

- $V_O(y, y_s)$ is a positive definite convex function such that the unique minimizer of $\min_{y \in \mathcal{Y}_s} V_O(y, y_s)$ is y_s.
- Optimization problem $P_N(x, y_s)$ given by:

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\min_{\mathbf{u}} & V_N^t(x, y_s; \mathbf{u}) \\
s.t. & x(0) = x, \\
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Properties

- **Feasibility**: The constraint $x(N) = x(N-1)$ defines an admissible equilibrium point, such that $x(N) = x(N-1) = Ax(N-1) + Bu(N-1)$.
- **Large domain of attraction**: Since the set of constraints of $P_N(x, y_s)$ does not depend on y_s, the feasible region of $P_N(x, y_s)$ does not depend on y_s either.
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- This controller integrates the economic cost function as a stationary extra cost added to the dynamic setpoint-tracking cost function

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Gradient based MPC: subRTO+MPC

- Low-cost formulation of the previous one. Instead of the nonlinear cost f_{eco}, the gradient of this function is added as extra cost to the MPC controller.

$$
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V_N^s(x, \hat{\mathbf{u}}, p; \mathbf{u}) \; = & \sum_{j=0}^{N-1} \|x(j) - x(N-1)\|_Q^2 + \|u(j) - u(N-1)\|_R^2 \\
& + \nabla f_{eco_{(\hat{x}(N-1), \hat{u}(N-1), p)}} \begin{bmatrix} x(N-1) - \hat{x}(N-1) \\ u(N-1) - \hat{u}(N-1) \end{bmatrix}
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where $(\hat{\mathbf{u}}, \hat{x}(N-1), \hat{u}(N-1))$ at time k is a previously known feasible solution, obtained using the shifted solution applied to the system at the sample time $k-1$.

Properties

- The MPC strategy is suboptimal:
  The control law at time k is derived from $\mathbf{u}(k) = \lambda \mathbf{u}^* + (1-\lambda)\hat{\mathbf{u}}$, $\lambda \in (0,1)$, which is a convex combination of $\hat{\mathbf{u}}$ with $\mathbf{u}^*$, being the last one the solution of the optimization problem $P_N^s(x,p)$.
- It however ensures recursive feasibility and convergence to the economic steady state, with a reduced computational cost.
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Economic MPC - EMPCT

- The economic MPC considers the economic cost function as the dynamic stage cost

$$V_N^e(x, y_s, p; \mathbf{u}) = \sum_{j=0}^{N-1} f_{eco}(x(j) - x(N-1) + x_s, u(j) - u(N-1) + u_s, p) + V_O(y(N-1), y_s)$$

- Optimization problem $P_N^e(x, y_s, p)$ given by:

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Properties

- Function $V_O(y(N{-}1), y_s)$ is the same as in the MPCT.
- As in the MPCT, we need to know the value of the economic setpoint (x_s, u_s, y_s), which means that we need to solve the RTO problem prior to the MPC problem.
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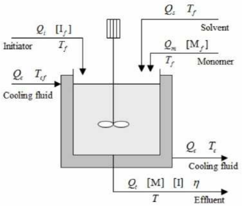
Polymerization reactor - CSTR



$I \xrightarrow{f_i, k_d} 2R$ (initiator decomposition)

$M + R \xrightarrow{k_i} P_i$ (chain initiation)

$P_n + M \xrightarrow{k_p} P_{n+1}$ (propagation)

$P_n + P_m \xrightarrow{k_{td}} T_n + T_m$ (termination by disproportionation)

$P_n + P_m \xrightarrow{k_{tc}} T_{n+m}$ (termination by combination)

1. Highly nonlinear system.
2. Output: the polymer intrinsic viscosity $y_1 = \eta$, and the reactor temperature $y_2 = T$.
3. Input: The controller manipulates the initiator flow-rate ($u_1 = Q_i$) and the liquid flow rate of the cooling jacket ($u_2 = Q_c$). The remaining inlet flow-rates Q_s and Q_m are related to Q_i by ratio control.

$$Q_m = \frac{\bar{Q}_m}{\bar{Q}_i} Q_i, \quad Q_s = 1.5 Q_m - Q_i$$

Linear model for prediction

$$A = \begin{bmatrix} 0.9788 & 0.0292 & 0.0010 \\ -0.0006 & 0.9375 & 0.0011 \\ 0.0124 & -0.0327 & 0.9569 \end{bmatrix},$$

$$B = \begin{bmatrix} 10.2205 & -1.2333 \\ -6.6059 & -1.3983 \\ -7.1717 & 0.2481 \end{bmatrix}, \quad C = \begin{bmatrix} -0.0757 & 0.0447 & -0.1073 \\ 0.6023 & -0.2749 & -0.0256 \end{bmatrix}$$

- Model obtained by subspace identification techniques.
- Linearization point: $u_{ss} = (0.030; 0.131)$ and $y_{ss} = (3.8968; 323.56)$.
- The PRBS signal used to excite the system has an amplitude of $0.1 u_{ss}$.
- The control scheme is been equipped with a state observer and disturbance estimator of the form:

$$\begin{aligned} \hat{x}^+ &= A\hat{x} + Bu + L_x(C\hat{x} + \hat{d} - y_p) \\ \hat{d}^+ &= \hat{d} + L_d(C\hat{x} + \hat{d} - y_p) \end{aligned}$$

where y_p is the output from the plant.

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Economic function and constraints

- Economic objective:maximization of the production rate plus a separation cost

$$f_{eco} = Q_t D_1 + (p(1) Q_c T - p(2) Q_c)$$

where $p = (p(1), p(2))$ are prices.

- Constraints: $\mathcal{X} = \{x \mid \|x\|_\infty \leq 20\}$, $\mathcal{U} = \{y \mid [0.01; 0.08]' \leq y \leq [0.07; 0.25]'\}$, and $\mathcal{Y} = \{y \mid [3; 310]' \leq y \leq [5.5; 331]'\}$.
- Simulations start at the nominal operating point (u_{ss}, y_{ss}). Three changes of prices have been considered: $p_1 = (1; 1)$, $p_2 = (1.5; 1)$ and $p_3 = (0.1; 5)$.

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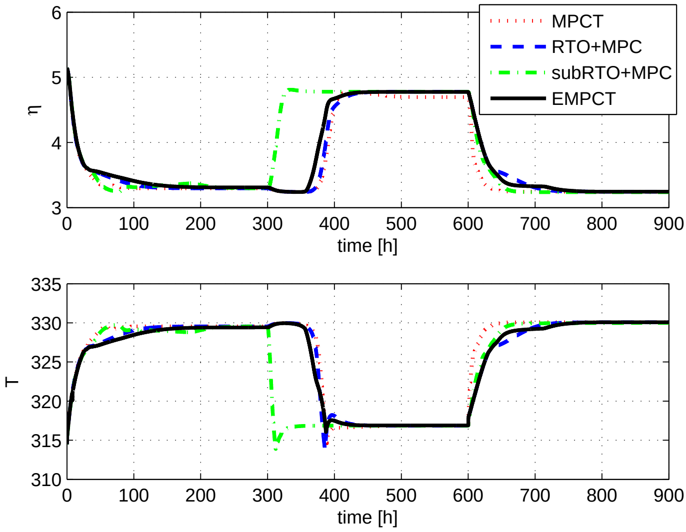
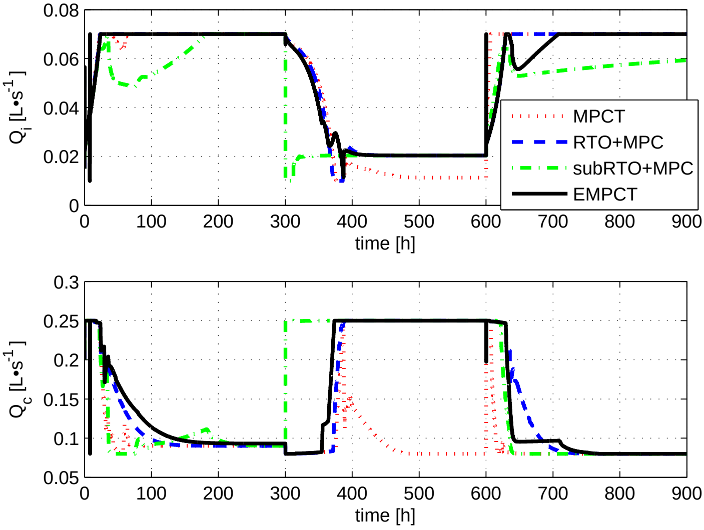
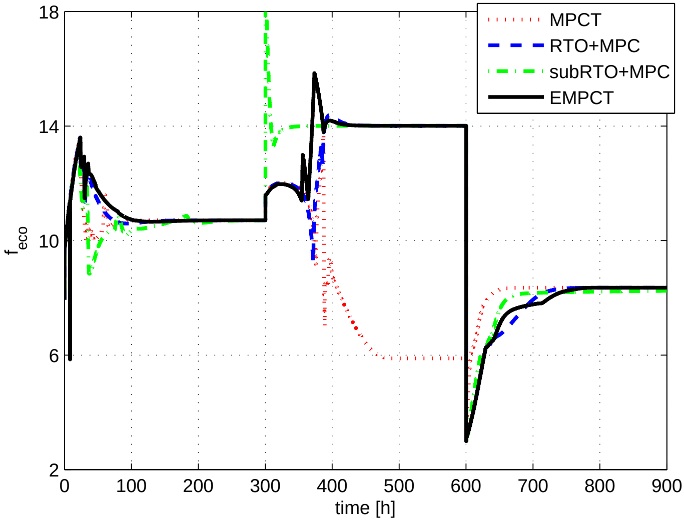
Economic optimality (1)



Economic optimality (2)



Economic optimality (3)



Economic optimality (4)

Analysis:

- All the controllers drive the system to the economically optimal setpoint, guaranteed feasibility and ensuring stability.
- EMPCT has a transient response very different from the other three controllers, because it optimizes the economics in the dynamic part of the MPC cost function. Hence the evolution of the system under the EMPCT controller is also optimal in the transient.
- subRTO+MPC speeds up the convergence to the economic setpoint.
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Computational burden

Table: Execution time (in seconds)

	Algo.	Max	Min	Average
MPCT	QP	0.4591	0.0096	0.0230
RTOMPC	SQP	0.6880	0.0182	0.0807
sub-RTOMPC	QP	0.6114	0.0065	0.0096
EMPCT	SQP	0.9382	0.0257	0.1393

- The MPCT is the faster algorithm since it needs to solve just a QP problem.
- The subRTO+MPC is also very fast in the QP solution. However, we are not considering the calculation of the gradient time.
- The solution of the EMPCT problem is clearly the one that needs more computational time, due to the high nonlinearity of the cost function.

Summary

- Each of the considered approaches is capable to ensure convergence, feasibility and stability, always fulfilling constraints.
- Setpoint tracking controllers speed up convergence to the setpoint.
- Economic MPC also provide economically optimal transient trajectories.
- Computational burden is also strictly connected with the proper formulation, with RTO+MPC and EMPCT being the most computational expensive, due to the nonlinearities in their formulations.
- MPCT and EMPCT needs to know the economic setpoint, hence an *a priori* RTO problem is needed to be solved.
- RTO+MPC and subRTO+MPC are able to drive the system to the economically optimal setpoint by themselves.

Main references

Adetola, V., & Guay, M. (2010). Integration of real-time optimization and model predictive control. *Journal of Process Control*, *20*(2), 125–133.

Alamo, T., Ferramosca, A., González, A. H., Limon, D., & Odloak, D. (2014). A gradient-based strategy for the one-layer RTO+MPC controller. *Journal of Process Control*, *24*(4), 435–447.

Amrit, R. (2011). *Optimizing Process Economics in Model Predictive Control*. Ph.D. thesis, University of Wisconsin, Madison.

Angeli, D., Amrit, R., & Rawlings, J. B. (2012). On average performance and stability of economic model predictive control. *IEEE Trans. on Automatic Control*, *57*(7), 1615–1626.

Biegler, L. T. (2009). Technology advances for dynamic real-time optimization. *Computer Aided Chemical Engineering - 10th International Symposium on Process Systems Engineering*, *27*, 1–6.

Diehl, M., Amrit, R., & Rawlings, J. B. (2011). A Lyapunov function for Economic Optimizing Model Predictive Control. *IEEE Transactions on Automatic Control*, *56*(3), 703 –707.

Engell, S. (2007). Feedback control for optimal process operation. *Journal of Process Control*, *17*(3), 203–219.

Ferramosca, A., Limon, D., Alvarado, I., Alamo, T., & Camacho, E. F. (2009). MPC for tracking with optimal closed-loop performance. *Automatica*, *45*(8), 1975–1978.

Ferramosca, A., Limon, D., & Camacho, E. F. (2014). Economic MPC for a Changing Economic Criterion for Linear Systems. *IEEE Transactions on Automatic Control*, *59*(10), –.

Limon, D., Alvarado, I., Alamo, T., & Camacho, E. F. (2008). MPC for tracking of piece-wise constant references for constrained linear systems. *Automatica*, *44*(9), 2382–2387.

Marchetti, A. G., .Ferramosca, A., & González, A. H. (2014). Steady-state target optimization designs for integrating real-timeoptimization and model predictive control. *Journal of Process Control*, *24*(1), 129–145.

Muske, K. (1997). Steady-state target optimization in linear model predictive control. In *Proceedings of the ACC*.

Qin, S. J., & Badgwell, T. A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, *11*(7), 733–764.

Würth, L., Rawlings, J. B., & Marquardt, W. (2009). Economic dynamic real-time optimization and nonlinear model predictive control on infinite horizons. In *Proceedings of the International Symposium on Advanced Control of Chemical Process*. Istanbul, Turkey.

Ying, C. M., & Joseph, B. (1999). Performance and stability analysis of LP-MPC and QP-MPC cascade control systems. *AIChE Journal*, *45*(7), 1521–1534.

Zanin, A. C., de Gouva, M. T., & Odloak, D. (2002). Integrating real time optimization into the model predictive controller of the FCC system. *Control Engineering Practice*, *10*(8), 819–831.

Thanks for your attention!