Grupo de Control Avanzado y Monitoreo de Procesos

INTEC - CONICET

Antonio Ferramosca

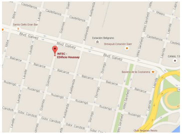
Università del Salento
Lecce, 22 Settembre 2015.

"Please, allow me to introduce myself..."

"Please, allow me to introduce myself..." (2)

Advanced Process Control and Monitoring Group

INTEC - CONICET.
Calle Güemes 3450 (3000). Santa Fe, Argentina



Researchers:

- Jacinto Marchetti, *Full Researcher - Head of the group*.
- Alejandro H. González, *Full Researcher*.
- Antonio Ferramosca, *Assistant Researcher*.
- José Luis Godoy, *Assistant Researcher*.

Postdoc:

- German Bustos.

Ph.D. Students:

- Agustina D'Jorge.
- Alejandro Anderson.

Research and projects

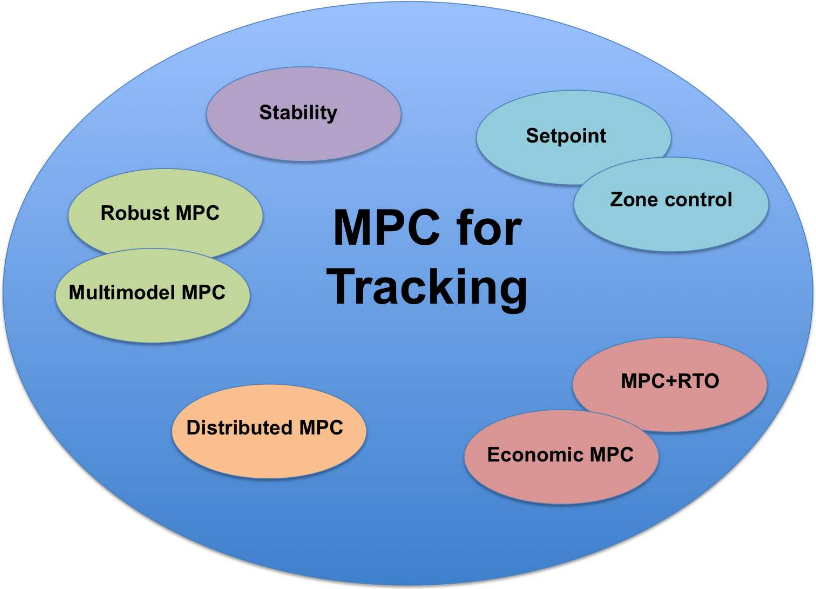
Research topics

1. MPC for tracking.
2. Economic MPC.
3. Distributed MPC.
4. MPC stability.
5. Robust and Stochastic MPC.
6. Closed-loop re-idetification.
7. Impulsive systems control $\rightarrow$ HIV drugs delivery control.
8. Monitoring and statistic control.

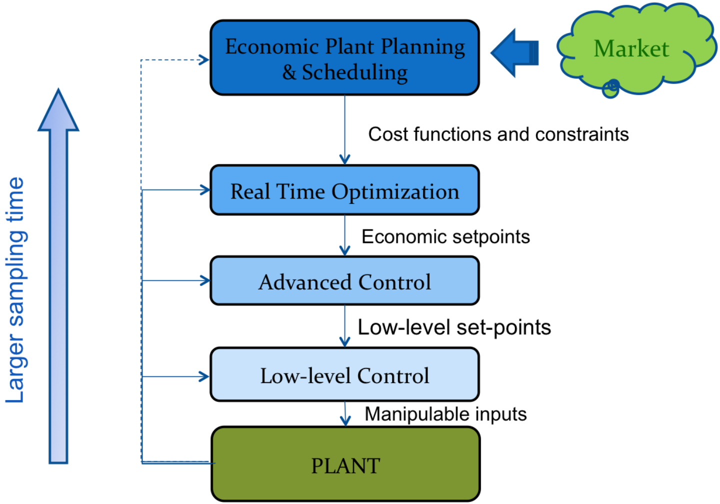
Industrial collaborations

- YPF S.A.

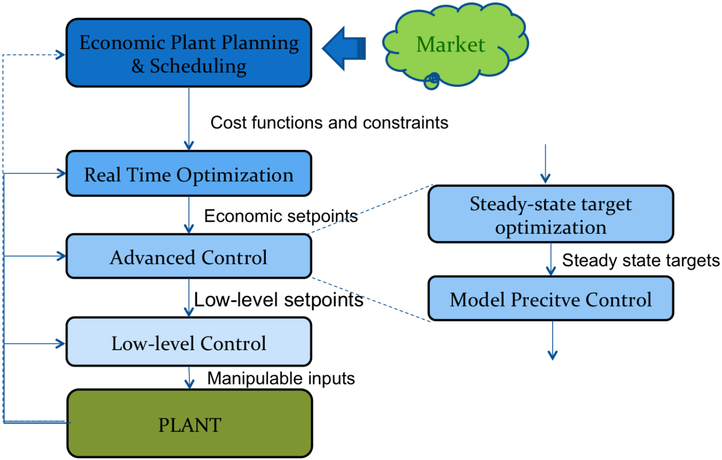
Research and projects (2)



Motivation


Larger sampling time
Economic Plant Planning & Scheduling
Market
Cost functions and constraints
Real Time Optimization
Economic setpoints
Advanced Control
Low-level set-points
Low-level Control
Manipulable inputs
PLANT

Motivation


Economic Plant Planning & Scheduling
Market
Cost functions and constraints
Real Time Optimization
Economic setpoints
Advanced Control
Low-level setpoints
Low-level Control
Manipulable inputs
PLANT
Steady-state target optimization
Steady state targets
Model Precitve Control

Model Predictive Control

Consider a discrete LTI system

$$x(k+1) = Ax(k) + Bu(k), \quad x(k) \in \mathcal{X},\ u(k) \in \mathcal{U}$$

At any k we want to find a control sequence

$$\mathbf{u} = \{u(k), u(k+1), ..., u(k+N-1)\}$$

that minimizes the performance index:

$$V_N(x; \mathbf{u}) = \sum_{j=0}^{N-1} (\|x(k+j)\|_Q^2 + \|u(k+j)\|_R^2) + \|x(k+N)\|_S^2$$

- $Q \geq 0,\ R > 0,\ S \geq 0.$
- Prediction horizon: $N \geq 1$

Model Predictive Control

Open-loop solution: minimize the performance index with respect to the control sequence **u**.

- OCP can be easily solved by means of a QP.

How do we close the loop? $\rightarrow$ Receding Horizon Principle

Definition (Receding Horizon Principle)

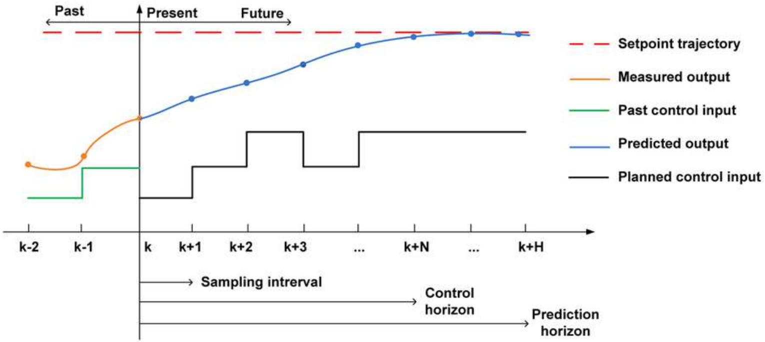
"at any time k solve the OCP over the prediction horizon [k,k+N] and apply only the first input $u^0(k)$ of the optimal sequence $\mathbf{u}^0(k)$. At time k+1, move the prediction window one step ahead, and repeat the optimization over the prediction horizon [k+1,k+N+1]"

Model Predictive Control

Open-loop solution:minimize the performance index with respect to the control sequence **u**.

- OCP can be easily solved by means of a QP.

How do we close the loop? $\rightarrow$ Receding Horizon Principle



Model Predictive Control

Stability (Mayne et al., 2000; Rawlings & Mayne, 2009):

- Terminal equality constraint.
- Terminal cost.
- Terminal inequality constraint.
- Terminal cost + Termina inequality constraint.

Terminal cost + Termina inequality constraint:

1. The terminal cost is positive definite and decreasing.
   - Take $S = P$, with P solution of DARE

$$P - (A + BK)' P (A + BK) = Q + K' RK$$

2. The terminal constraint is an invariant set.
   - Ω is an invariant set for the LTI system if, for any $x(k) \in \Omega$, there exists $u(k)$ s. t. $Ax(k) + Bu(k) \in \Omega$.

Based on this, the optimal performance index can be considered as a Lyapunov function

Lyapunov Function

1. $\alpha_1(x(k)) \leq V_N^0(x(k)) \leq \alpha_2(x(k)), \quad \forall x(k) \in \mathcal{X}$
2. $V_N^0(x(k+1)) - V_N^0(x(k)) \leq -\alpha_3(x(k))$.

$\alpha_1, \alpha_2, \alpha_3$ are $\mathcal{K}_\infty$ functions.

(Aleksandr Lyapunov (1857-1918))

Recall the performance index:

$$V_N(x;\mathbf{u}) = \sum_{j=0}^{N-1} (\|x(k+j)\|_Q^2 + \|u(k+j)\|_R^2) + \|x(k+N)\|_P^2$$

1. It is positive definite.
2. It can be proved that

$$V_N^0(x(k+1)) - V_N^0(x(k)) \leq -(\|x(k)\|_Q^2 + \|u^0(k)\|_R^2)$$

So the origin is an A-stable equilibrium.

RTO problem

The MPC controller should steer the system to the economically optimal reachable steady state:

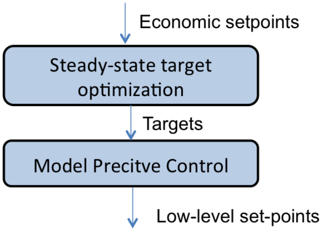
Definition

The economically optimal reachable steady state and input, (x_s^*, u_s^*), satisfy

$$
\begin{aligned}
(x_s^*, u_s^*) \quad &= \quad \arg\min_{x,u} \ell(x,u) \\
s.t. \quad & x = f(x,u) \\
& x \in \mathcal{X}, \quad u \in \mathcal{U}
\end{aligned}
$$

MPC for tracking

Steady State Optimizer: For a given RTO set-point (x_s^*, u_s^*), computes an admissible steady state target for the MPC (x_{sp}, u_{sp}), typically minimizing a quadratic error function (also knonw as QP-MPC structure).



Standard Tracking MPC (Muske & Rawlings, 1993)

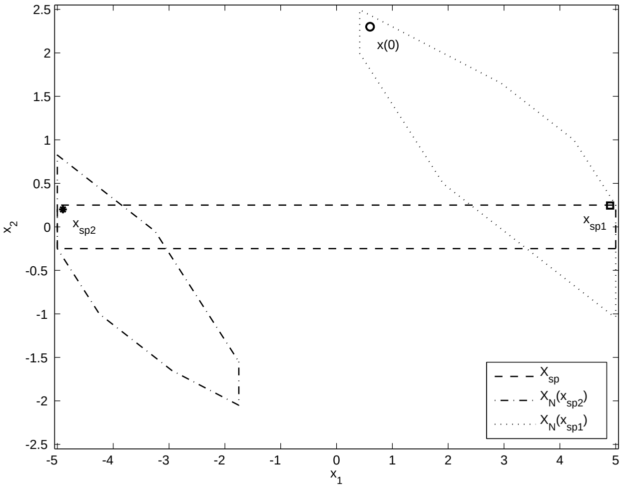
$$\min_{\mathbf{u}} \sum_{j=0}^{N-1} (\|x(k+j) - x_{sp}\|_Q^2 + \|u(k+j) - u_{sp}\|_R^2) + \|x(k+N) - x_{sp}\|_P^2$$

$$\begin{aligned} s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\ & x(k+N) \in \Omega \end{aligned}$$

- K stabilizing gain. P solution of DARE.
- Ω invariant set around x_{sp}.

Loss of feasibility

If the target changes, the feasibility may be lost and the MPC must be redesigned.



Loss of feasibility

Some existing solutions to this problem:

- Translation to the new setpoint (*Muske and Rawlings*)
- MPC with an infinite horizon (*Constrained LQR*)
- Reference Governors (*Gilbert*)
- Dual mode strategy for tracking (*Chisci and Zappa*)
- Model Predictive Control with slack variables (*Odloak*)
- Considering the change of reference as a disturbance to be rejected (*Pannocchia and Kerrigan*)

Loss of feasibility

Some existing solutions to this problem:

- Translation to the new setpoint (*Muske and Rawlings*)
- MPC with an infinite horizon (*Constrained LQR*)
- Reference Governors (*Gilbert*)
- Dual mode strategy for tracking (*Chisci and Zappa*)
- Model Predictive Control with slack variables (*Odloak*)
- Considering the change of reference as a disturbance to be rejected (*Pannocchia and Kerrigan*)
- MPC for tracking (*Limon and Ferramosca*)

MPC for tracking

Standard MPC VS. MPC for tracking

$$\begin{array}{ll} \min\limits_{\mathbf{u}} & \sum\limits_{j=0}^{N-1} (\|x(k+j) - x_{sp}\|_Q^2 + \|u(k+j) - u_{sp}\|_R^2) + \|x(k+N) - x_{sp}\|_P^2 \\ s.t. & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\ & x(k+N) \in \Omega \end{array}$$

$$\begin{array}{ll} \min\limits_{\mathbf{u}, x_a, u_a} & \sum\limits_{j=0}^{N-1} (\|x(k+j) - x_a\|_Q^2 + \|u(k+j) - u_a\|_R^2) + \|x(k+N) - x_a\|_P^2 + V_O(x_a, x_{sp}) \\ s.t. & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\ & x_a = Ax_a + Bu_a \\ & (x(k+N), x_a, u_a) \in \Omega_t^a \end{array}$$

MPC for tracking

$$\begin{aligned}
\min_{\mathbf{u},x_a,u_a} \quad & \sum_{j=0}^{N-1} (\|x(k+j)-x_a\|_Q^2 + \|u(k+j)-u_a\|_R^2) + \|x(k+N)-x_a\|_P^2 + V_O(x_a, x_{sp}) \\
s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\
& x_a = Ax_a + Bu_a \\
& (x(k+N), x_a, u_a) \in \Omega_t^a
\end{aligned}$$

- Ω_t^a is an invariant set for tracking. That is, for all $(x, x_a, u_a) \in \Omega_t^a$

$$\begin{aligned}
& (x, K(x-x_a)+u_a) \in \mathcal{X} \times \mathcal{U} \\
& x_a = Ax_a + Bu_a \\
& (Ax + B(K(x-x_a)+u_a), x_a, u_a) \in \Omega_t^a
\end{aligned}$$

- The offset-cost function $V_O(x_a, x_{sp})$ is convex, positive definite, with $V_O(0,0) = 0$, and such that the minimizer of

$$\min_x V_O(x, x_{sp})$$

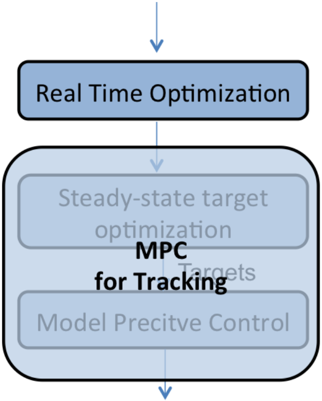
is unique.

MPC for tracking

- Feasibility: it is always guaranteed for any x_{sp}.
- Stability: cost function is a Lyapunov function.
- Local optimality: cost function as optimal as in standard MPC.

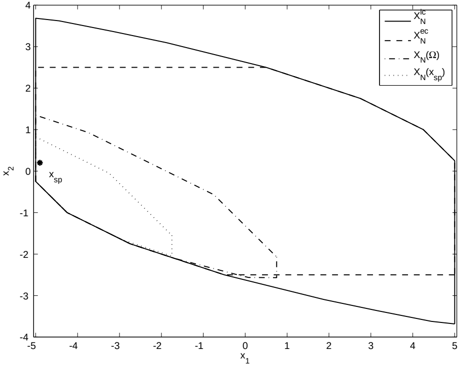
MPC for tracking

- Feasibility: it is always guaranteed for any x_{sp}.
- Stability: cost function is a Lyapunov function.
- Local optimality: cost function as optimal as in standard MPC.
- $V_O(x_a, x_{sp})$ is an SSTO.



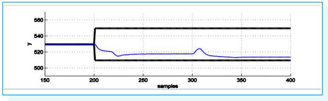
MPC for tracking

- Feasibility: it is always guaranteed for any x_{sp}.
- Stability: cost function is a Lyapunov function.
- Local optimality: cost function as optimal as in standard MPC.
- $V_O(x_a, x_{sp})$ is an SSTO.
- Larger domain of attraction.



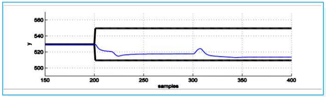
Zone control MPC for tracking

- Output in a range: exact value not important.
- Optimal economic steady state given by ranges.
- Typical in industries: more outputs then inputs.



Zone control MPC for tracking

- Output in a range: exact value not important.
- Optimal economic steady state given by ranges.
- Typical in industries: more outputs then inputs.



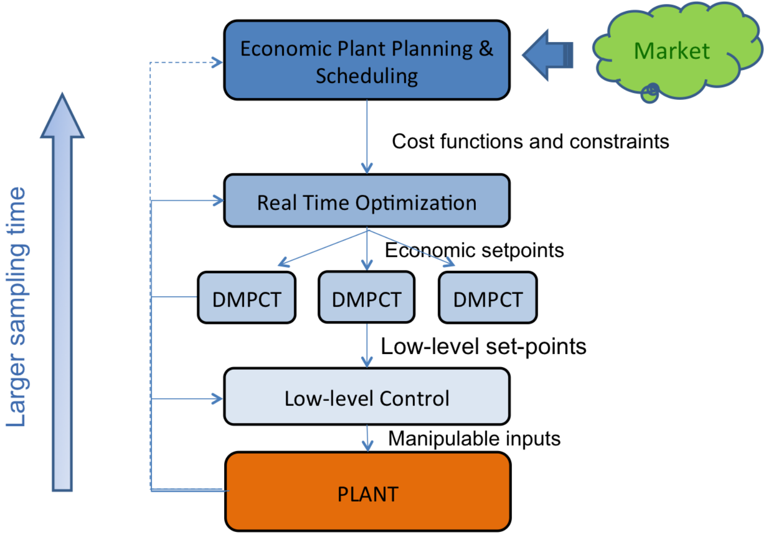
$$\min_{\mathbf{u},x_a,u_a} \sum_{j=0}^{N-1}(\|x(k+j)-x_a\|_Q^2+\|u(k+j)-u_a\|_R^2)+\|x(k+N)-x_a\|_P^2+V_O(x_a,\Gamma_{sp})$$

$$\begin{aligned} s.t. \quad & x(k+j)\in\mathcal{X},\ u(k+j)\in\mathcal{U},\ j\in\mathbb{I}_{[0,N-1]} \\ & x_a = Ax_a + Bu_a \\ & (x(k+N),x_a,u_a)\in\Omega_t^a \end{aligned}$$

- Γ_{sp} is a polyhedron.
- If $x\in\Gamma_{sp}$, then $V_O(x,\Gamma_{sp})=0$, otherwise $V_O(x,\Gamma_{sp})>0$.

Cooperative MPC for tracking

"*agents share open-loop information in order to improve closed-loop performance*". (Rawlings & Mayne, 2009).



Distributed partition of the plant

- Original system partitioned in M subsystems of the form:

$$
\begin{aligned}
x_i^+ &= A_i x_i + \sum_{j=1}^{M} \bar{B}_{ij} u_j \\
y_i &= C_i x_i + \sum_{j=1}^{M} \bar{D}_{ij} u_j
\end{aligned}
$$

  where $x_i \in \mathbb{R}^{n_i}$, $u_j \in \mathbb{R}^{m_j}$, $y_i \in \mathbb{R}_i^{p}$, $A_i \in \mathbb{R}^{n_i \times n_i}$, $\bar{B}_{ij} \in \mathbb{R}^{n_i \times m_j}$, $C_i \in \mathbb{R}^{p_i \times n_i}$ and $\bar{D}_{ij} \in \mathbb{R}^{p_i \times m_j}$.

- Without loss of generality, it is considered that

$$u = (u_1, \cdots, u_M)$$

- For the two players game:

$$
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^+ = \begin{bmatrix} A_1 & \\ & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \bar{B}_{11} \\ \bar{B}_{21} \end{bmatrix} u_1 + \begin{bmatrix} \bar{B}_{12} \\ \bar{B}_{22} \end{bmatrix} u_2
$$

Cooperative MPC for tracking

Players share a common objective: overall plant cost function.

$$V_N(x, x_{sp}; \mathbf{u}, x_a, u_a) = \sum_{j=0}^{N-1} (\|x(j) - x_a\|_Q^2 + \|u(j) - u_a\|_R^2) + \|x(N) - x_a\|_P^2 + V_O(x_a, x_{sp})$$

Features:

- Artificial steady state and input as decision variables.
- Penalizing the deviation of the predicted trajectory with the artificial steady conditions.
- Centralized offset-cost function.

Optimization problem (2 players game)

Each *i*-th agent solves an iterative decentralized optimization problem

$$
\begin{aligned}
(\mathbf{u}_i^*, x_a^*, u_a^*) \; &= \; \arg \min_{\mathbf{u}_i, x_{a,i}, u_{a,i}} V_t(x, x_{sp}; \mathbf{u}, x_a, u_a) \\
s.t. \;\; & x_q(j+1) = A_q x_q(j) + \sum_{\ell=1}^{2} B_{q\ell} u_\ell(j), \;\; q \in \mathbb{I}_{1,2} \\
& x(0) = (x_1, x_2), \\
& \mathbf{u}_\ell = \mathbf{u}_\ell^{[p]}, \quad \ell \in \mathbb{I}_{1,2} \setminus i, \\
& x(j) \in \mathcal{X}, \\
& (u_1(j), u_2(j)) \in \mathcal{U}, \quad j = 0, ..., N-1 \\
& (x(N), x_a, u_a) \in \Omega_t^a
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{u}_1^{[p+1]} &= w_1 \mathbf{u}_1^* + w_2 \mathbf{u}_1^{[p]} \\
\mathbf{u}_2^{[p+1]} &= w_1 \mathbf{u}_2^{[p]} + w_2 \mathbf{u}_2^* \\
x_a^{[p+1]} &= w_1 x_{a,1}^*(x, \mathbf{u}_2^{[p]}) \\
&\quad + w_2 x_{a,2}^*(x, \mathbf{u}_1^{[p]}) \\
u_a^{[p+1]} &= w_1 u_{a,1}^*(x, \mathbf{u}_2^{[p]}) \\
&\quad + w_2 u_{a,2}^*(x, \mathbf{u}_1^{[p]}) \\
w_1 + w_2 &= 1 \quad w_1, w_2 > 0
\end{aligned}
$$

Stop:

- $\mathbf{u}_1^{[p+1]} = \mathbf{u}_1^{[p]}$
- $p = \bar{p}$

- Centralized invariant set for tracking.
- Simultaneous solution (Bertsekas & Tsitsiklis, 1997, pp. 219-223) and convex update.
- Iterate $(\mathbf{u}_1^{[0]}, \mathbf{u}_2^{[0]})$: *warm-start* algorithm.

The *warm start* algorithm

The *warm start* algorithm

1. First candidate:

$$\tilde{\mathbf{u}}(k+1)=\{u(1;k),...,u(N-1;k),u_c(N)\}$$

where

$$u_c(N) = K(x(N) - x_a^*(k)) + u_a^*(k)$$

centr. terminal control law.

The *warm start* algorithm

1. First candidate:

$$\tilde{\mathbf{u}}(k+1)=\{u(1;k),...,u(N-1;k),u_c(N)\}$$

where

$$u_c(N) = K(x(N) - x_a^*(k)) + u_a^*(k)$$

centr. terminal control law.

2. Second candidate:

$$\hat{\mathbf{u}}(k+1)=\{\hat{u}_c(0),...,\hat{u}_c(N-1)\}$$

where

$$\begin{aligned} \hat{x}(0) &= x(k+1) \\ \hat{x}(j+1) &= A\hat{x}(j)+BK(\hat{x}(j)-\hat{x}_a^*(k)) \\ &\quad +Bu_a^*(k),\ j \in \mathbb{I}_{1:N-2} \\ \hat{u}_c(j) &= K(\hat{x}(j)-x_a^*(k))+u_a^*(k) \end{aligned}$$

The *warm start* algorithm

1. First candidate:

$$\tilde{\mathbf{u}}(k+1)=\{u(1;k),...,u(N-1;k),u_c(N)\}$$

where

$$u_c(N) = K(x(N) - x_a^*(k)) + u_a^*(k)$$

centr. terminal control law.

2. Second candidate:

$$\hat{\mathbf{u}}(k+1)=\{\hat{u}_c(0),...,\hat{u}_c(N-1)\}$$

where

$$\begin{aligned}\hat{x}(0) &= x(k+1)\\ \hat{x}(j+1) &= A\hat{x}(j)+BK(\hat{x}(j)-\hat{x}_a^*(k))\\ &\quad +Bu_a^*(k),\ j\in\mathbb{I}_{1:N-2}\\ \hat{u}_c(j) &= K(\hat{x}(j)-x_a^*(k))+u_a^*(k)\end{aligned}$$

3. **IF** $(x(k+1), x_a^*(k), u_a^*(k)) \in \Omega_t^a$ **AND** $V_N(x(k+1), x_{sp}, \hat{\mathbf{u}}) \leq V_N(x(k+1), x_{sp}, \tilde{\mathbf{u}})$
   SET
$$\mathbf{u}(k+1)^{[0]} = (\hat{\mathbf{u}}(k+1), x_a^*(k), u_a^*(k))$$
   ELSE
$$\mathbf{u}(k+1)^{[0]} = (\tilde{\mathbf{u}}(k+1), x_a^*(k), u_a^*(k))$$

Properties

Stability: closed-loop A-stable.

- Convexity.
- Recursive feasibility: warm start and centralized terminal constraint.
- Sub-optimal MPC (Scokaert et al., 1999).

Properties

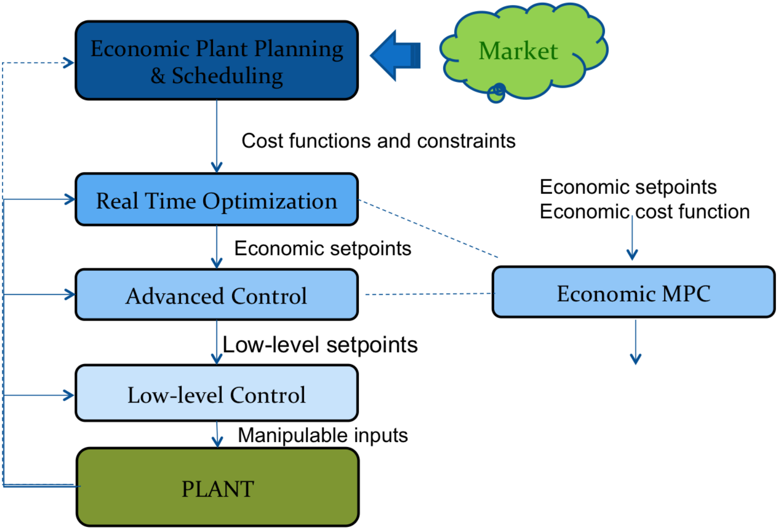
Stability: closed-loop A-stable.

- Convexity.
- Recursive feasibility: warm start and centralized terminal constraint.
- Sub-optimal MPC (Scokaert et al., 1999).

Optimality: closed-loop converges to centralized optimum.

- For coupled and uncoupled constraints. For any $\bar{p} \geq 1$.
- Centralized offset cost function (SSTO).
- Warm start.

Motivation


Economic Plant Planning & Scheduling
Market
Cost functions and constraints
Real Time Optimization
Economic setpoints
Economic cost function
Economic setpoints
Advanced Control
Economic MPC
Low-level setpoints
Low-level Control
Manipulable inputs
PLANT

Economic steady state

The economic controller should steer the system to the optimal reachable steady state:

Definition

The optimal reachable steady state and input, (x_s^*, u_s^*), satisfy

$$
\begin{aligned}
(x_s^*, u_s^*) \quad &= \quad \arg\min_{x,u} \ell(x,u) \\
s.t. \quad & x = f(x,u) \\
& x \in X, \quad u \in U
\end{aligned}
$$

- $\ell(x,u)$ is usually highly nonlinear and may be negative at some point.

One layer RTO+MPC: motivation (Zanin et al., 2002)

Integrate the RTO optimization into the MPC layer (Petrobras).

$$
\begin{aligned}
\min_{\mathbf{u}, x_a, u_a} \quad & \sum_{j=0}^{N-1} (\|x(k+j) - x_a\|_Q^2 + \|u(k+j) - u_a\|_R^2) + \ell(x_a, u_a) \\
s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\
& x_a = Ax_a + Bu_a \\
& x(k+N) = x_a
\end{aligned}
$$

One layer RTO+MPC: motivation (Zanin et al., 2002)

Integrate the RTO optimization into the MPC layer (Petrobras).

$$\begin{aligned} \min_{\mathbf{u}, x_a, u_a} \quad & \sum_{j=0}^{N-1} (\|x(k+j) - x_a\|_Q^2 + \|u(k+j) - u_a\|_R^2) + \ell(x_a, u_a) \\ s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\ & x_a = Ax_a + Bu_a \\ & x(k+N) = x_a \end{aligned}$$

- Economic (stationary) and dynamic objectives in one optimization problem: no need to calculate (x_s^*, u_s^*).
- Direct extension of the MPC for tracking.
- Better economic performance: reduced inconsistency/unreachability.

One layer RTO+MPC: motivation (Zanin et al., 2002)

Integrate the RTO optimization into the MPC layer (Petrobras).

$$
\begin{aligned}
\min_{\mathbf{u}, x_a, u_a} \quad & \sum_{j=0}^{N-1} (\|x(k+j) - x_a\|_Q^2 + \|u(k+j) - u_a\|_R^2) + \ell(x_a, u_a) \\
s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\
& x_a = Ax_a + Bu_a \\
& x(k+N) = x_a
\end{aligned}
$$

- Economic (stationary) and dynamic objectives in one optimization problem: no need to calculate (x_s^*, u_s^*).
- Direct extension of the MPC for tracking.
- Better economic performance: reduced inconsistency/unreachability.
- Drawback:highly nonlinear optimization problem.

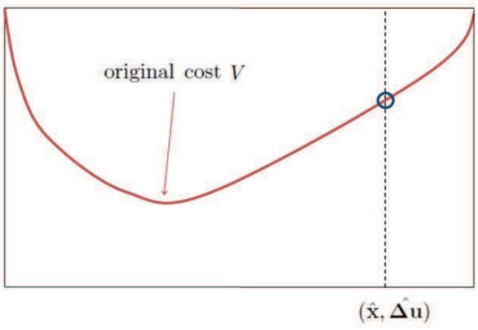
Gradient based One-layer RTO+MPC

Solve an approximated problem, instead of the original one.

(1) Define a feasible solution $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$ to the original problem

Feasible solution $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$

$$\hat{\mathbf{u}} = \{\hat{u}_0, \hat{u}_1, \cdots, \hat{u}_{N-1}\}, \quad \hat{\mathbf{x}} = \{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_a\}$$



Gradient based One-layer RTO+MPC (2)

(2) Define an approximated MPC problem, $P_N^{app}(x)$, at the feasible point $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$

Problem $P_N^{app}(x)$

$$
\begin{array}{ll}
\min\limits_{\mathbf{u}} & V_N^{app}(x, \hat{x}_a, \hat{u}_a; \mathbf{u}, x_a, u_a) \\
s.t. & x(k+j) \in \mathcal{X},\; u(k+j) \in \mathcal{U},\; j \in \mathbb{I}_{[0,N-1]} \\
 & x_a = Ax_a + Bu_a \\
 & x(k+N) = x_a
\end{array}
$$

Gradient based One-layer RTO+MPC (2)

(2) Define an approximated MPC problem, $P_N^{app}(x)$, at the feasible point $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$

Problem $P_N^{app}(x)$

$$
\begin{array}{ll}
\min\limits_{\mathbf{u}} & V_N^{app}(x, \hat{x}_a, \hat{u}_a; \mathbf{u}, x_a, u_a) \\
s.t. & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\
 & x_a = Ax_a + Bu_a \\
 & x(k+N) = x_a
\end{array}
$$

Approximated MPC Costs at $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$

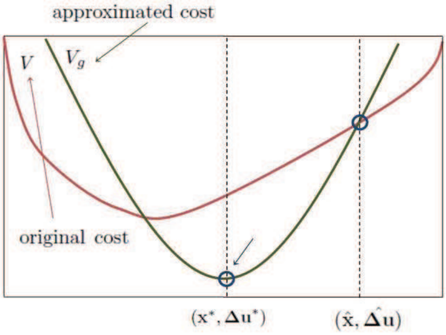
$$
\begin{aligned}
V_N^{app}(x, \hat{x}_a, \hat{u}_a; \mathbf{u}, x_a, u_a) \;&=\; \sum_{j=0}^{N-1} (\|x(j) - x_a\|_Q^2 + \|u(j) - u_a\|_R^2) + \ell(\hat{x}_a, \hat{u}_a) \\
&+\; \nabla \ell(\hat{x}_a, \hat{u}_a) \begin{bmatrix} x_a - \hat{x}_a \\ u_a - \hat{u}_a \end{bmatrix}
\end{aligned}
$$

Gradient based One-layer RTO+MPC (3)

(3) Compute the optimal solution $(\mathbf{x}^*, \mathbf{u}^*, x_a^*, u_a^*)$ to the approximated problem

Solution $(\mathbf{x}^*, \mathbf{u}^*, x_a^*, u_a^*)$

$$\mathbf{u}^* = \{u_0^*, u_1^*, \cdots, u_{N-1}^*\}, \quad \mathbf{x}^* = \{x_1^*, x_2^*, \cdots, x_a^*\}$$



Gradient based One-layer RTO+MPC (4)

(4) Define a convex combination of $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$ and $(\mathbf{x}^*, \mathbf{u}^*, x_a^*, u_a^*)$

- Convex combination

$$\begin{aligned} \mathbf{u}(\lambda) &= (1-\lambda)\hat{\mathbf{u}} + \lambda \mathbf{u}^*, \quad \mathbf{x}(\lambda) = (1-\lambda)\hat{\mathbf{x}} + \lambda \mathbf{x}^* \\ x_a(\lambda) &= (1-\lambda)\hat{x}_a + \lambda x_a^*, \quad u_a(\lambda) = (1-\lambda)\hat{u}_a + \lambda u_a^*, \quad \lambda \in [0,1] \end{aligned}$$

Gradient based One-layer RTO+MPC (4)

(4) Define a convex combination of $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$ and $(\mathbf{x}^*, \mathbf{u}^*, x_a^*, u_a^*)$

- Convex combination

$$\begin{aligned} \mathbf{u}(\lambda) &= (1-\lambda)\hat{\mathbf{u}} + \lambda \mathbf{u}^*, \quad \mathbf{x}(\lambda) = (1-\lambda)\hat{\mathbf{x}} + \lambda \mathbf{x}^* \\ x_a(\lambda) &= (1-\lambda)\hat{x}_a + \lambda x_a^*, \quad u_a(\lambda) = (1-\lambda)\hat{u}_a + \lambda u_a^*, \quad \lambda \in [0,1] \end{aligned}$$

- Original and Approx. cost, $V_N(x; \mathbf{u}, x_a, u_a)$, $V_N^{app}(x; \mathbf{u}, x_a, u_a)$, parameterized in λ

$$\begin{aligned} V(\lambda) &= \sum_{j=0}^{N-1} (\|x_j(\lambda) - x_a(\lambda)\|_Q^2 + \|u_j(\lambda) - u_a(\lambda)\|_R^2) + \ell(x_a(\lambda), u_a(\lambda)) \\ V_g(\lambda) &= \sum_{j=0}^{N-1} (\|x_j(\lambda) - x_a(\lambda)\|_Q^2 + \|u_j(\lambda) - u_a(\lambda)\|_R^2) + \ell(\hat{x}_a, \hat{u}_a) \\ &\quad + \nabla \ell(\hat{x}_a, \hat{u}_a) \begin{bmatrix} x_a(\lambda) - \hat{x}_a \\ u_a(\lambda) - \hat{u}_a \end{bmatrix} \end{aligned}$$

Gradient based One-layer RTO+MPC (5)

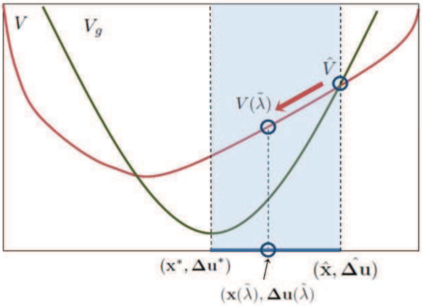
- Extreme cases

$$\hat{V} \triangleq V(\lambda = 0); \quad V_g(\lambda = 0) = \hat{V}$$

$$V^* \triangleq V(\lambda = 1); \quad V_g(\lambda = 1) \text{ optimal value and so } V_g(1) < V_g(0)$$

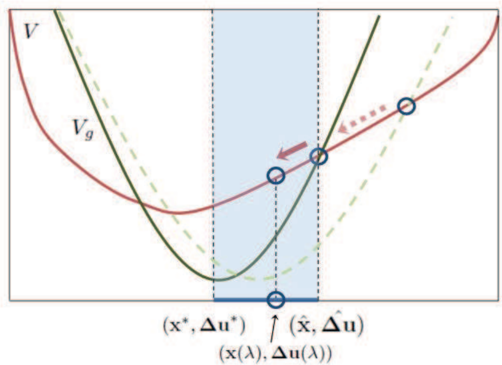
Main Theorem

1. $(\mathbf{x}(\lambda), \mathbf{u}(\lambda), x_a(\lambda), u_a(\lambda))$, for $\lambda \in [0, 1]$, is a feasible solution to the original problem.
2. There exists a $\tilde{\lambda} \in (0, 1]$ such that $V(\tilde{\lambda}) < V(0) = \hat{V}$



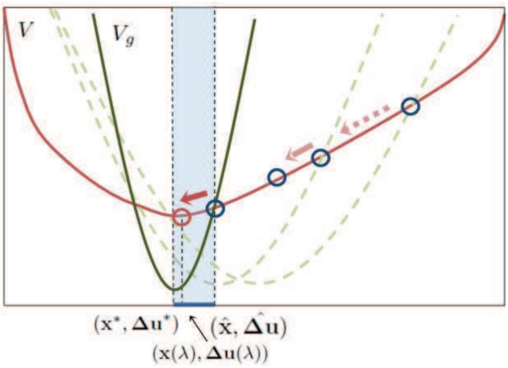
Gradient based One-layer RTO+MPC (6)

(5) Using $(\mathbf{x}(\tilde{\lambda}), \mathbf{u}(\tilde{\lambda}), x_a(\tilde{\lambda}), u_a(\tilde{\lambda}))$ as a new feasible point, a decreasing $V_N(x)$ is obtained



Gradient based One-layer RTO+MPC (7)

(6) Stability, recursive feasibility, and convergence to x_s^* are ensured



Proposed Algorithm

Algorithm

At each sample time k,

1. *compute the feasible solution* $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$
2. *compute the gradient of the economic cost function* $\ell(x, u)$ *w.r.t.* (x, u), $\nabla\ell(x, u)$
3. *compute the optimal solution to the approx. problem,* $(\mathbf{x}^*, \mathbf{u}^*, x_a^*, u_a^*)$
4. *compute the value of the parameter* $\tilde{\lambda}$*, of the main theorem*
5. *compute the convex combination*
6. *take the first input action from the solution* $\mathbf{u}(\tilde{\lambda})$ *to implement the MPC*

Proposed Algorithm

Algorithm

At each sample time k,

1. *compute the feasible solution* $(\hat{\mathbf{x}}, \hat{\mathbf{u}}, \hat{x}_a, \hat{u}_a)$
2. *compute the gradient of the economic cost function* $\ell(x, u)$ *w.r.t.* (x, u), $\nabla\ell(x, u)$
3. *compute the optimal solution to the approx. problem,* $(\mathbf{x}^*, \mathbf{u}^*, x_a^*, u_a^*)$
4. *compute the value of the parameter* $\tilde{\lambda}$*, of the main theorem*
5. *compute the convex combination*
6. *take the first input action from the solution* $\mathbf{u}(\tilde{\lambda})$ *to implement the MPC*

Remark

- *The implemented control action is not provided by an optimization solution*
- *One can heuristically search for a value of* λ *that gives a cost* $V(\lambda)$ *smaller than* $\hat{V}$
- *Within the current sample time the solution* $(\mathbf{x}(\tilde{\lambda}), \mathbf{u}(\tilde{\lambda}), x_a(\tilde{\lambda}), u_a(\tilde{\lambda}))$ *can be iteratively improved*

Economic MPC: motivation (Rawlings et al., 2012)

Contest

- *The closer the system gets to the economic optimum, the more profitable it is.*
- *Who gets closest to the global economic optimum?*
  - *Tracking controllers: Rush to the target (away from non steady economic optimum).*
  - *Tracking speed chosen through penalties, but still the objective remains to drive away from non steady economic optimum!*
- *Economic optimizing controllers: Expected to get closer to the economic optimum with eventual setting at the steady target.*

Economic MPC: formulation

Economic MPC control law derived from the solution of the optimization problem

$$
\begin{aligned}
\min_{\mathbf{u}} \quad & \sum_{j=0}^{N-1} \ell(x(k+j), u(k+j)) \\
s.t. \quad & \\
& x(k+j) \in X, \; u(k+j) \in U, \quad j \in \mathbb{I}_{0:N-1} \\
& x(k+N) = x_s^*
\end{aligned}
$$

- Economic MPC cost not positive definite.
- Stability cannot be prove by means of Lyapunov arguments.

Economic MPC: stability

Definition

Dissipativity (Angeli et al., 2012)
A control system $f(x,u)$ is dissipative with respect to a supply rate $s : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$ if there exists a function $\lambda : \mathcal{X} \to \mathbb{R}$ such that

$$\lambda(f(x,u)) - \lambda(x) \leq s(x,u)$$

for all $(x,u) \in \mathcal{X} \times \mathcal{U}$. If in addition $\rho : \mathcal{X} \to \mathbb{R}_{\geq 0}$ positive definite exists such that

$$\lambda(f(x,u)) - \lambda(x) \leq -\rho(x) + s(x,u)$$

then the system is said to be strictly dissipative.

In Economic MPC context, if one takes

$$s(x,u) = \ell(x,u) - \ell(x_s^*, u_s^*)$$

then x_s^* is an asymptotically stable equilibrium point of the closed-loop system.

Economic MPC: stability (2)

Defining a *rotated* stage cost function as:

$$L_r(x,u) = \ell(x,u) + \lambda(x) - \lambda(f(x,u))$$

A-stability of x_s^* can be proved resorting to Lyapunov arguments.

Assumption (Particular case: Strong duality (Diehl et al., 2011))

Let $L_r(x,u)$ be the rotated stage cost function given by

$$L_r(x,u) = \ell(x,u) + \lambda'(x - f(x,u)) - \ell(x_s^*, u_s^*)$$

where λ is a multiplier that ensures that the rotated cost exhibits a unique minimum at (x_s^*, u_s^*) for all $x \in \mathcal{X}$, $u \in \mathcal{U}$. Then there exist two $\mathcal{K}$-functions α and such that $L_r(x,u) \geq \alpha(|x - x_s^*|)$.

- Difficult to verify in general, but...

Economic MPC: stability (2)

Defining a *rotated* stage cost function as:

$$L_r(x,u) = \ell(x,u) + \lambda(x) - \lambda(f(x,u))$$

A-stability of x_s^* can be proved resorting to Lyapunov arguments.

Assumption (Particular case: Strong duality (Diehl et al., 2011))

Let $L_r(x,u)$ be the rotated stage cost function given by

$$L_r(x,u) = \ell(x,u) + \lambda^{'}(x - f(x,u)) - \ell(x_s^*, u_s^*)$$

where λ is a multiplier that ensures that the rotated cost exhibits a unique minimum at (x_s^*, u_s^*) for all $x \in \mathcal{X}$, $u \in \mathcal{U}$. Then there exist two $\mathcal{K}$-functions α and such that $L_r(x,u) \geq \alpha(|x - x_s^*|)$.

- Difficult to verify in general, but...
- ... always fulfilled for linear systems and convex functions.

What if the economic criterion changes?

- Consider an economic criterion $\ell(x, u, p)$ with p economic parameter.
- If p changes, x_s^* also changes: feasibility may be lost.

What if the economic criterion changes?

- Consider an economic criterion $\ell(x, u, p)$ with p economic parameter.
- If p changes, x_s^* also changes: feasibility may be lost.

$$
\begin{aligned}
\min_{\mathbf{u}, x_a, u_a} \quad & \sum_{j=0}^{N-1} \ell(x(k+j) - x_a + x_s^*, u(k+j) - u_a + u_s^*) + V_O(x_a, x_s^*) \\
s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\
& x_a = Ax_a + Bu_a \\
& x(k+N) = x_a
\end{aligned}
$$

What if the economic criterion changes?

- Consider an economic criterion $\ell(x, u, p)$ with p economic parameter.
- If p changes, x_s^* also changes: feasibility may be lost.

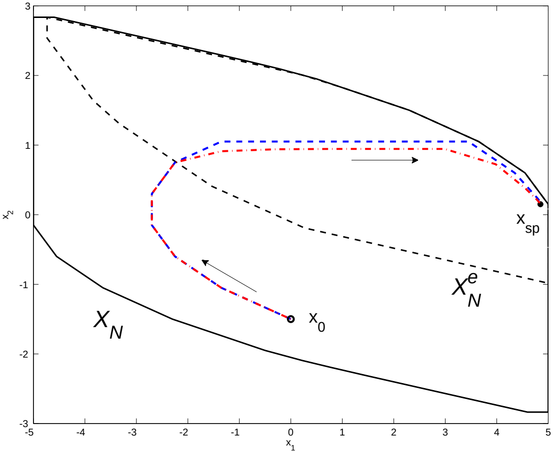
$$\begin{aligned}
\min_{\mathbf{u}, x_a, u_a} \quad & \sum_{j=0}^{N-1} \ell(x(k+j) - x_a + x_s^*, u(k+j) - u_a + u_s^*) + V_O(x_a, x_s^*) \\
s.t. \quad & x(k+j) \in \mathcal{X},\ u(k+j) \in \mathcal{U},\ j \in \mathbb{I}_{[0,N-1]} \\
& x_a = Ax_a + Bu_a \\
& x(k+N) = x_a
\end{aligned}$$

- Closed-loop converges to x_s^*.
- The rotated cost $L_r(x(k+j) - x_a + x_s^*, u(k+j) - u_a + u_s^*)$ is a Lyapunov function.
- No need to calculate λ... but x_s^* is still needed.

EMPC for changing economic criterion

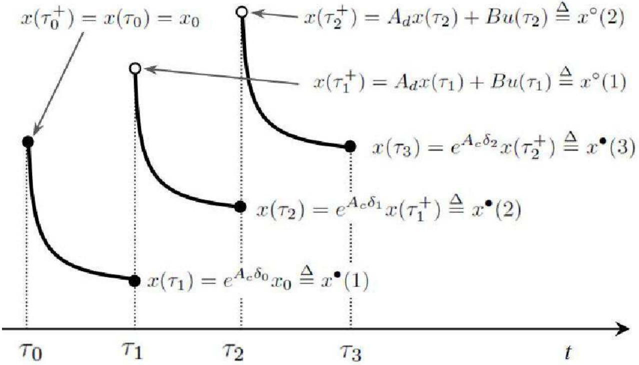
Properties:

- Guaranteed feasibility as MPCT.
- Economic optimality as EMPC.



Impulsive system: motivation

- Continuous dynamics but discrete actuation.
- Main motivation: drugs delivery for HIV treatment.



Impulsive system

- Let a linear impulsive first-order differential equations of the form

$$\begin{cases} \dot{x}(t) &= A_c x(t), \quad x(0) = x_0, \quad t \neq \tau_k, \\ x(\tau_k^+) &= A_d x(\tau_k) + Bu(\tau_k), \quad k \in \mathbb{N}, \end{cases}$$

  where $t \in \mathbb{R}$, $x \in \mathcal{X} \subseteq \mathbb{R}^n$, $u \in \mathcal{U} \subseteq \mathbb{R}^m$ (impulsive control).

- The only equilibrium of the impulsive system (1) is given by $(u_s, x_s) = (0, 0)$.
- Both, $\mathcal{X}$ and $\mathcal{U}$ are compact sets and contain the origin in their interior.

Assumption

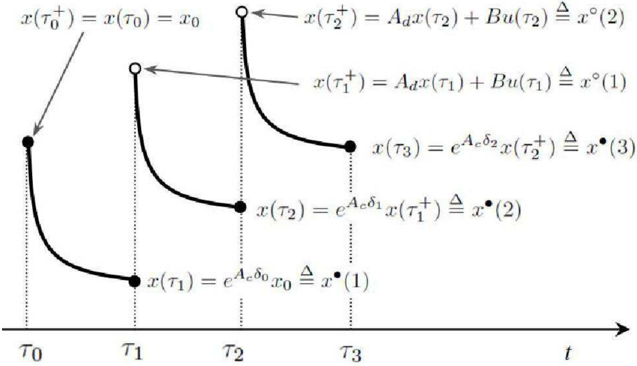
Let us denote the initial time as $t_0 = 0$ and the set of time instants as $\mathcal{T} = \{0, \tau_1, \cdots, \tau_k, \cdots\}$, with δ being $\delta = \tau_{i+1} - \tau_i$.

Underlying discrete time systems

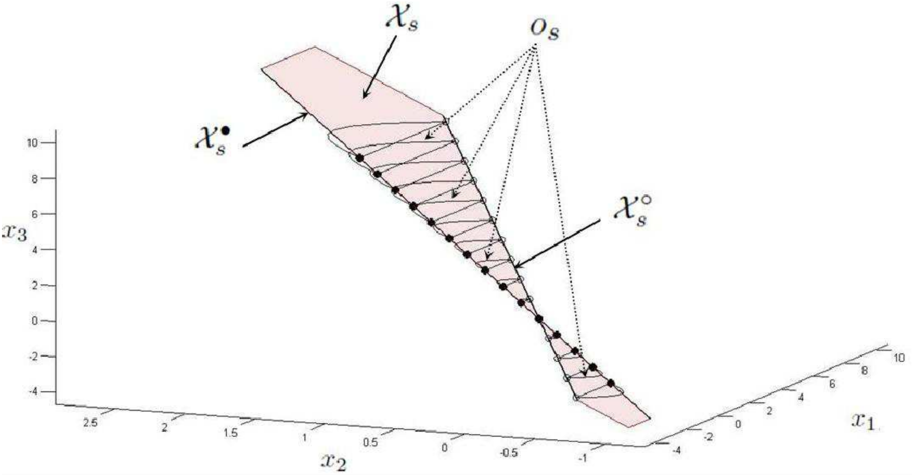
- Underlying discrete time systems

$$\begin{aligned} x^\bullet(j+1) &= A^\bullet x^\bullet(j) + B^\bullet u^\bullet(j), \quad x^\bullet(0) = x_0, \\ x^\circ(j+1) &= A^\circ x^\circ(j) + B^\circ u^\circ(j), \quad x^\circ(0) = x_0, \end{aligned}$$

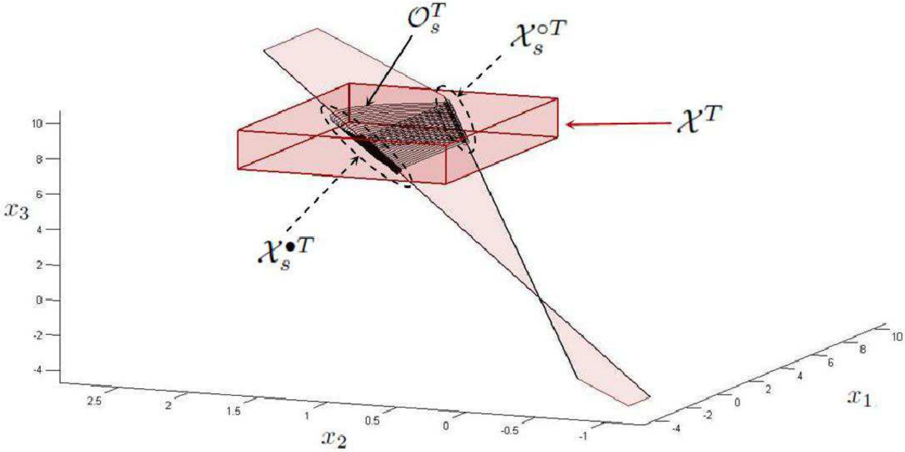
where $A^\circ = T = A_d e^{A_c \delta}$, $A^\bullet = T^* = e^{A_c \delta} A_d$, $B^\circ = B$, $B^\bullet = e^{A_c \delta} B$, and $u^\circ(j+1) = u^\bullet(j)$, for $j \geq 0$.



Cone $\mathcal{X}_s$ into a subspace of dimension 2 in $\mathbb{R}^3$, sets $\mathcal{X}_s^\circ$ and $\mathcal{X}_s^\bullet$, and equilibrium orbits o_s of a simulated impulsive system



MPC for tracking a therapeutic window



Proposed MPC strategy

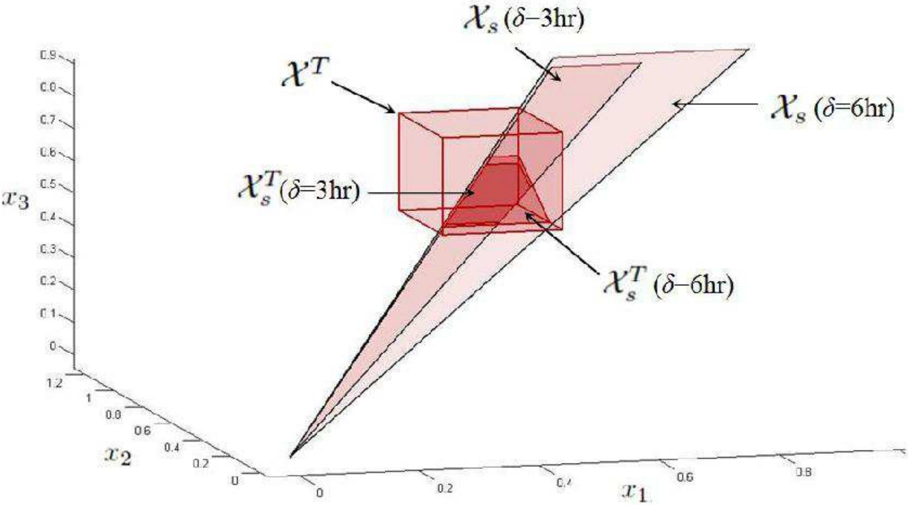
- It is a zone MPCT problem.
- The control objective is to drive the system from a given initial state x_0 to an equilibrium objective set defined by $\mathcal{X}_s^{\bullet T} \subset \mathcal{X}^T$.
- The MPC cost is given by:

$$V(x, p; \mathbf{u}, x_a, u_a) = \overbrace{\sum_{j=0}^{N-1} \|x(j) - x_a\|_Q^2 \|u(j) - u_s\|_R^2}^{\text{dyanimc cost}} + \overbrace{p\left(dist(x_a, \mathcal{X}_s^{\bullet T}) + dist(u_a, \mathcal{U}_s^T)\right)}^{\text{terminal cost}}$$

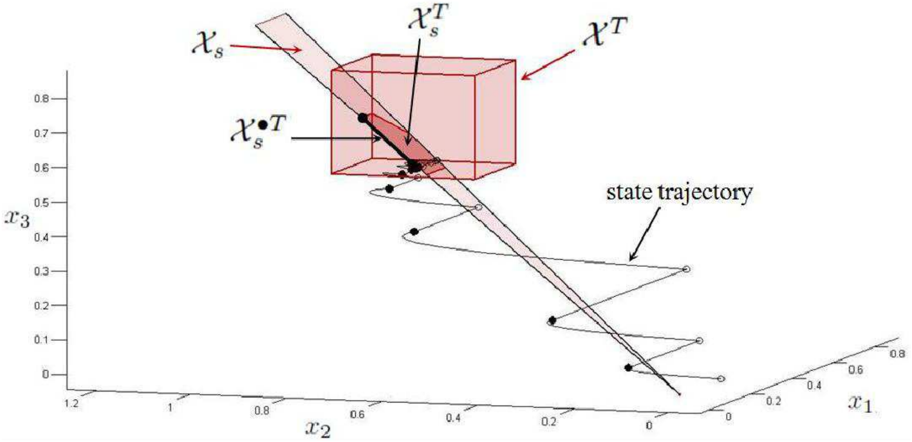
with $Q > 0$, $R > 0$ and $p > 0$.
- The current state x and the sets $\mathcal{X}_s^{\bullet T}$ and $\mathcal{U}_s^T$ are parameters, while $(\mathbf{u}, x_a, u_a)$ are the optimization variables (being N the control horizon).
- The optimization problem to be solved at time k by the MPC is given by P_{MPC}:

$$\begin{aligned}
\min_{\mathbf{u}, x_a, u_a} \quad & V(x, p; \mathbf{u}, x_a, u_a) \\
s.t. \quad & \\
& x(0) = x, \\
& x(j+1) = A^\bullet x(j) + B^\bullet u(j), \qquad j \in \mathbb{I}_{0:N-1} \\
& x(j) \in \mathcal{X},\ u(j) \in \mathcal{U}, \qquad j \in \mathbb{I}_{0:N-1} \\
& x(N) = x_a, \quad x_a = A^\bullet x_a + B^\bullet u_a
\end{aligned}$$

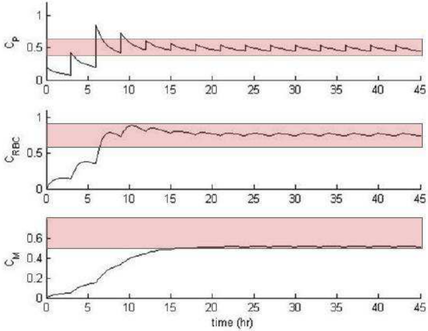
Case study. State equilibrium set $\mathcal{X}_s^T$ for $\delta = 3$ *hr* and $\delta = 6$ *hr*

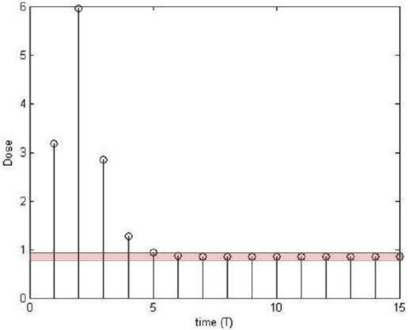


Case study. State-space evolution



Case study. State and Input time evolution





Acknowledgment

- Daniel Limon (*Universidad de Sevilla, Spain*).
- Alejandro H. González (*CONICET - Universidad Nacional del Litoral, Argentina*).
- Teodoro Alamo (*Universidad de Sevilla, Spain*).
- Darci Odloak (*Universidad de Sao Paul, Brazil*).
- James B. Rawlings (*University of Wisconsin - Madison, USA*).
- Pablo S. Rivadaneira (*CONICET - Universidad Nacional del Litoral, Argentina*).

Main references

Angeli, D., Amrit, R., & Rawlings, J. B. (2012). On average performance and stability of economic model predictive control. *IEEE Trans. on Automatic Control*, *57*(7), 1615–1626.

Bertsekas, D. P., & Tsitsiklis, J. N. (1997). *Parallel and Distributed Computation*. Athena Scientific, Belmont, Massachusetts.

Diehl, M., Amrit, R., & Rawlings, J. B. (2011). A Lyapunov function for Economic Optimizing Model Predictive Control. *IEEE Transactions on Automatic Control*, *56*(3), 703 –707.

Mayne, D. Q., Rawlings, J. B., Rao, C. V., & Scokaert, P. O. M. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, *36*(6), 789–814.

Muske, K., & Rawlings, J. B. (1993). Model predictive control with linear models. *AIChE Journal*, *39*, 262–287.

Rawlings, J. B., Angeli, D., & Bates, C. N. (2012). Fundamentals of Economic Model Predictive Control. In *Proceedings of the 51st IEEE Conference on Decision and Control (CDC)*, (pp. 3851–3861).

Rawlings, J. B., & Mayne, D. Q. (2009). *Model Predictive Control: Theory and Design*. Nob-Hill Publishing, 1st ed.

Scokaert, P. O. M., Mayne, D. Q., & Rawlings., J. B. (1999). Suboptimal model predictive control (feasibility implies stability). *IEEE Transactions on Automatic Control*, *44*(3), 648–654.

Zanin, A. C., de Gouva, M. T., & Odloak, D. (2002). Integrating real time optimization into the model predictive controller of the FCC system. *Control Engineering Practice*, *10*(8), 819–831.

Thanks for your attention!