One-layer robust MPC: a multi-model approach

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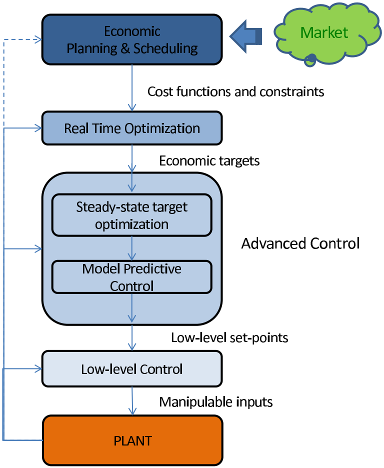
Outline

Real Time Optimization

Hierarchical control structure (Qin & Badgwell (2003); Engell (2007)):



1. The Real Time Optimizer (RTO) computes the operation point according to economic criteria and operation limits.
2. The RTO solves an optimization problem based on a complex nonlinear stationary model of the plant.
3. The setpoints computed by the RTO are sent to the MPC control.
4. MPC solves a QP based on a simplified dynamic model of the plant and constraints.

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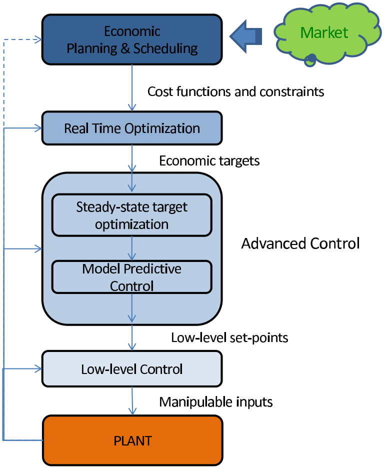

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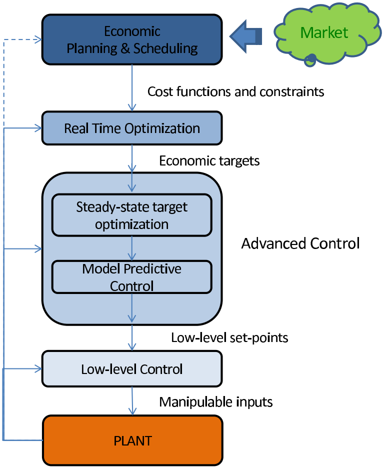

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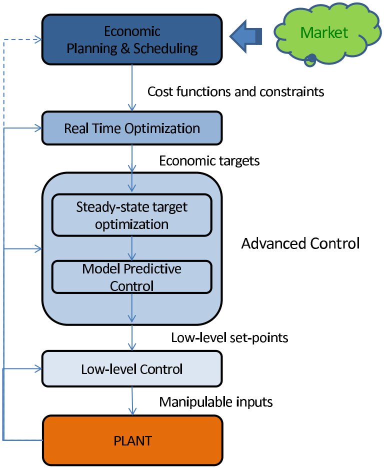

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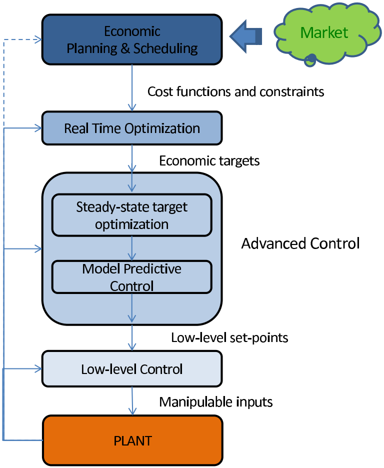

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Drawbacks of the control structure

- RTO time scale (hours or days) VS MPC time scale (minutes).
- Slow reaction to process variations (disturbances).
- There exist mismatches between the models of RTO and MPC.
- The RTO may provide inconsistent setpoints to the MPC.
  - Unreachability of the setpoints.
  - Poor economic performances.

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Existing solutions

- SSTO or LP/QP-MPC (Muske (1997); Ying & Joseph (1999); Marchetti et al. (2014))
  - Same model as the MPC.
  - Same sample time as the MPC.

- One-layer solutions
  - Dynamic RTO (Biegler (2009); Kadam & Marquardt (2007); Würth et al. (2009))
  - One layer RTO+MPC (Adetola & Guay (2010); Zanin et al. (2002))

- Economic MPC (Amrit (2011); Angeli et al. (2012); Diehl et al. (2011); Fagiano & Teel (2013); Ferramosca et al. (2014))

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Industrial application

- Controllers use linear prediction models.
- Plants has sparse operation points with different economics behaviors.
- When the operation point changes, the prediction model is not adequate to represents the new operative condition.
  - YPF (Argentina) and Petrobras (Brazil) report this issue.

Proposed solution:
multi-model representation of the plant (Badgwell (1997))

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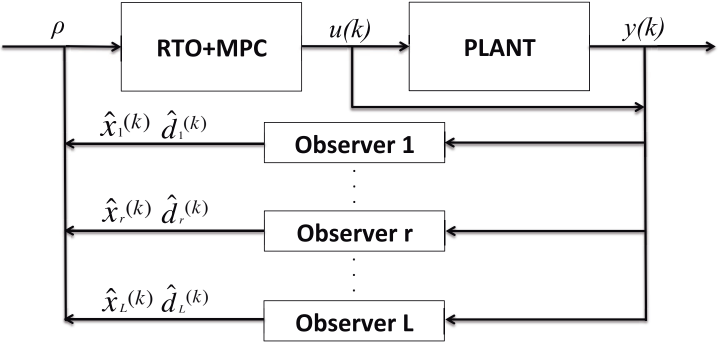
Multi-model description of the plant



- The real model of the plant is not known.
- We have a collection of linear model $\Pi = \{\pi_1, ..., \pi_L\}$ s.t.

$$\pi_i : \quad x_i^+ = A_i x + B_i u, \quad y_i = C_i x, \quad i \in \mathbb{I}_{1:L}$$

- $\pi_r \in \Pi$ defines the real model, $\pi_{no} \in \Pi$ a nominal model for predictions.

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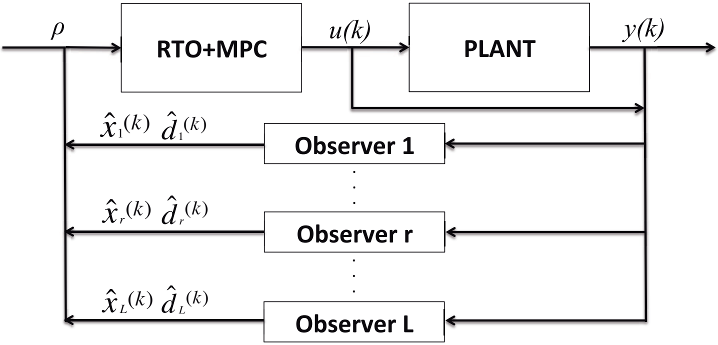

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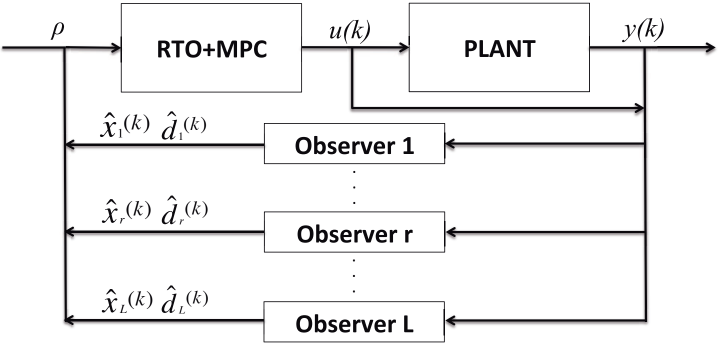

- Define an augmented system with an integrating output disturbance:

$$\begin{bmatrix} x_i^+ \\ d_i^+ \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} x_i \\ d_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u$$

$$y_i = \begin{bmatrix} C_i & I_p \end{bmatrix} \begin{bmatrix} x_i \\ d_i \end{bmatrix}$$

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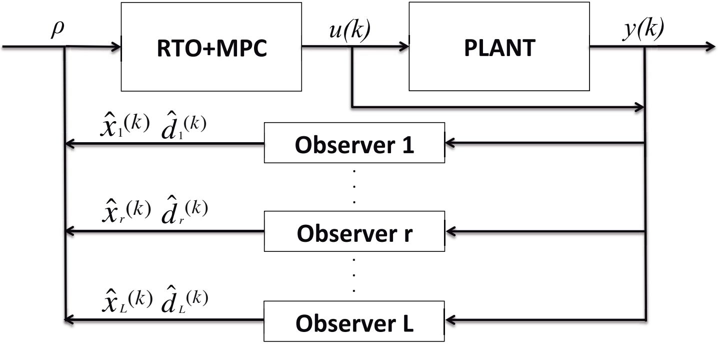

- Control structure equipped with one observer per each model:

$$\begin{aligned} \hat{x}_i(k+1) &= A_i\hat{x}(k) + B_i u(k) \\ \hat{d}_i(k+1) &= \hat{d}_i(k) + L_i^d(C_i\hat{x}_i(k) - y(k) + \hat{d}_i(k)) \end{aligned}$$

- $\hat{d}(k)$ is an estimation of the output disturbance.

Basic Assumptions

- $x(k) \in \mathcal{X}$, $u(k) \in \mathcal{U}$, for all $k \geq 0$. $\mathcal{X}$ is convex and closed, $\mathcal{U}$ is convex and compact and both sets contain the origin in their interior.

- A_i is stable. The pair (A_i, B_i) is controllable.

- $f_{eco}(y, u, \rho)$ is a convex nonlinear function that takes into account the economic objectives of the plant.

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(x_s^*, u_s^*, y_s^*) \quad &= \quad \arg \min_{(x,u,y)} f_{eco}(y, u, \rho) \\
s.t. \quad & x \in \mathcal{X}, \quad u \in \mathcal{U} \\
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- ρ is a parameter that takes into account prices, costs or production goals.

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Basic Assumptions

- The stationary conditions of each model can be mapped in a m-dimensional linear subspace s. t.

$$(x_{s,i}, u_s) = M_{\theta,i}\theta, \quad y_{s,i} = N_{\theta,i}\theta$$

given a parameter $\theta \in \mathbb{R}^m$.

- The sets of admissible equilibrium states, inputs and outputs are

$$\begin{aligned}
\mathcal{Z}_{s,i} &= \{(x_i, u) \in \mathcal{X} \times \mathcal{U} \mid x_i = A_i x_i + B_i u\} \\
\mathcal{X}_{s,i} &= \{x_i \in \mathcal{X} \mid \exists u \in \mathcal{U} \text{ such that } (x_i, u) \in \mathcal{Z}_{s,i}\} \\
\mathcal{Y}_{s,i} &= \{y = C_i x_i \mid (x_i, u) \in \mathcal{Z}_{s,i}\}
\end{aligned}$$

Multi-model one-layer MPC

Include the RTO economic objective in the MPC control problem.

- Economic and dynamic objectives in one optimization problem.
- Reduces incosistency/unreachability.
- Better economic performance.

The proposed cost function reads:

$$V_N(x,\hat{d},\rho;\mathbf{u},\theta)=\sum_{j=0}^{N-1}\|x_{no}(j)-x_{s,no}\|_Q^2+\|u(j)-u_s\|_R^2+V_O(h_s,u_s,\rho)$$

where

$$V_O(h_s,u_s,\rho)=\sum_{i=1}^{L}f_{eco}(y_{s,i},u_s,\rho),\ \text{and } h_s=\begin{bmatrix} y_{s,1}\\ \vdots \\ y_{s,L}\end{bmatrix}$$

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Optimization problem

Problem $P_N(x, \hat{z}, \hat{d}, \rho, \tilde{\mathbf{u}}, \tilde{\theta})$

$$
\begin{array}{lllr}
\min\limits_{\mathbf{u},\theta} & V_N(x, \hat{d}, \rho; \mathbf{u}, \theta) & & \\
s.t. & x_i(0) = \hat{x}_i, & & i \in \mathbb{I}_{1:L} \\
& x_i(j+1) = A_i x_i(j) + B_i u(j), & j \in \mathbb{I}_{0:N-1}, & i \in \mathbb{I}_{1:L} \\
& x_i(j) \in \mathcal{X},\ u(j) \in \mathcal{U}, & j \in \mathbb{I}_{0:N-1}, & i \in \mathbb{I}_{1:L} \\
& (x_{s,i}, u_s) = M_{\theta,i}\theta, & & i \in \mathbb{I}_{1:L} \\
& y_{s,i} = N_{\theta,i}\theta + \hat{d}_i, & & i \in \mathbb{I}_{1:L} \\
& x_i(N) = x_{s,i} & & i \in \mathbb{I}_{1:L} \\
& V_N^i(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}, \theta) \leq V_N^i(\hat{x}_i, \hat{d}_i, \rho; \tilde{\mathbf{u}}, \tilde{\theta}), & & i \in \mathbb{I}_{1:L}
\end{array}
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where

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V_N^i(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}, \theta) = \sum_{j=0}^{N-1} \|x_i(j) - x_{s,i}\|_Q^2 + \|u(j) - u_s\|_R^2 + f_{eco}(y_{s,i}, u_s, \rho)
$$

- $\tilde{\mathbf{u}}$ and $\tilde{\theta}$ are feasible solutions, based on a solution of the same problem at time $k-1$.
- Last constraint: robust stability constraint (Badgwell, 1997).

Stability

Theorem

For any ρ, the closed-loop system controlled by the proposed controller is stable and converges asymptotically to a steady point $(x_s^*, u_s^*, y_s^*) = \arg\min_{(x,u,y)} f_{eco}(y, u, \rho)$.

Sketch of the proof:

- Recursive feasibility is ensured by the fact that the real plant $\pi_r \in \Pi$.
- Cost decreasing is ensured by the robust constraint

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Properties of the proposed controller

- The controller ensures feasibility under any change of the economic objective.
- The controller ensures convergence to the equilibrium point provided by the RTO

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Simulation example

The 4 tanks plant

Interesting features:

1. All the states are measurable.
2. The outputs are strongly coupled.
3. The system is nonlinear.
4. The states and inputs are constrained.

$$\frac{dh_1}{dt} = -\frac{a_1}{A}\sqrt{2gh_1} + \frac{a_3}{A}\sqrt{2gh_3} + \frac{\gamma_a}{A}\frac{q_a}{3600}$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A}\sqrt{2gh_2} + \frac{a_4}{A}\sqrt{2gh_4} + \frac{\gamma_b}{A}\frac{q_b}{3600}$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A}\sqrt{2gh_3} + \frac{(1-\gamma_b)}{A}\frac{q_b}{3600}$$
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Simulation example

- Linearized model:

$$\frac{dx}{dt} = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{A}{A\tau_3} & 0 \\ 0 & \frac{-1}{\tau_2} & 0 & \frac{A}{A\tau_4} \\ 0 & 0 & \frac{-1}{\tau_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_a}{3600A} & 0 \\ 0 & \frac{\gamma_b}{3600A} \\ 0 & \frac{(1-\gamma_b)}{3600A} \\ \frac{(1-\gamma_a)}{3600A} & 0 \end{bmatrix} u$$

where $x_i = h_i - h_i^o$, $u_j = q_j - q_j^o$, $j = a, b$ and $i = 1, \cdots, 4$.

$\tau_i = \frac{A}{a_i}\sqrt{\frac{2h_i^0}{g}} \geq 0$, $i = 1, \cdots, 4$.

- Models:linearization points

Model	h_1	h_2	q_a	q_b
π_1	0.4210	0.4678	1.4802	1.5197
π_2	0.2977	0.3308	1.2447	1.2779
π_3	0.8550	0.5672	1.0444	2.6980
π_{no}	0.6487	0.6636	1.63	2

- Model π_{no} is the nominal model.
- Model π_1 is been taken as the real plant model.

Simulation example

- Economic cost function:

$$f_{eco}(y, u, \rho) = (q_a^2 + \rho_1 q_b^2) + \rho_2 \frac{V_{min}}{A(h_1 + h_2)}$$

  where $y = (h_1, h_2)$, $u = (q_a, q_b)$, $\rho = (\rho_1, \rho_2)$, and V_{min} is the minimum volume to be accumulated.

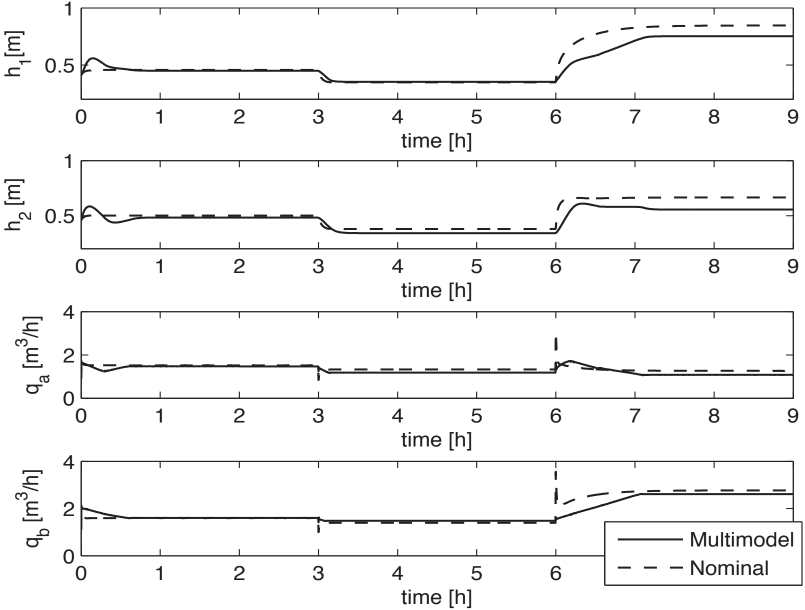
- $\rho^{[1]} = (1, 20)$, $\rho^{[2]} = (1, 10)$ and $\rho^{[3]} = (0.4, 30)$.
- MPC setup: $Q = I_4$, $R = 0.01 I_2$, and $N = 6$.
- Constraints:

$$\begin{aligned} 0.2 &\leq h_i \leq 1.20\,[m] \\ 0 &\leq q_a \leq 3.26\,[m^3/h] \\ 0 &\leq q_b \leq 4.00\,[m^3/h] \end{aligned}$$

- Simulations in Matlab using *fmincon*.

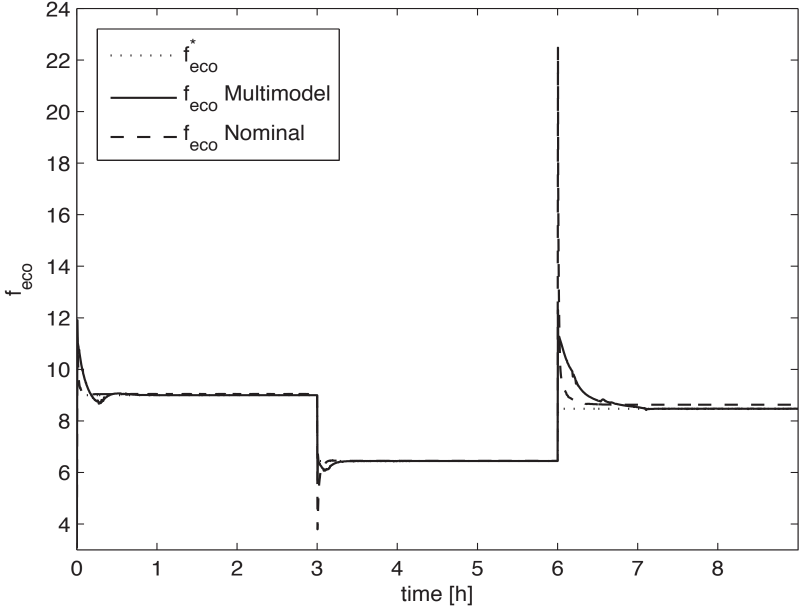
Simulation results

Outputs and inputs: time evolutions



Simulation results

Economic costs: time evolutions



Summary

Conclusions:

- A multi-model one layer RTO+MPC strategy has been proposed to reduce inconsistencies in the traditional hierarchical structure RTO/MPC.
- The multi-model approach ensures robustness to model uncertainties or changes in the operative conditions.
- Recursive feasibility and convergence are always ensured, for any changing economic objective.
- Stability is proved resorting to Lyapunov arguments.

Future works:

- Nonconvex f_{eco}.

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Thanks for your attention!