Distributed MPC for tracking.

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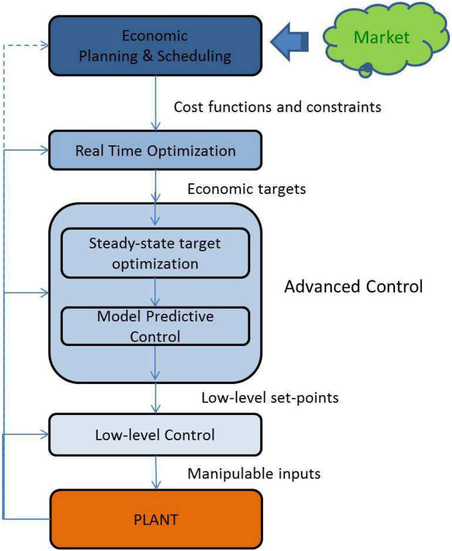
Outline

Large scale systems control structure



1. Market-oriented decision making.
2. RTO for the economic optimization.
3. Advanced control: MPC.
4. SSTO: extra optimization level. It reduces the gap between RTO and MPC.
5. Low level control and feedback from the plant.

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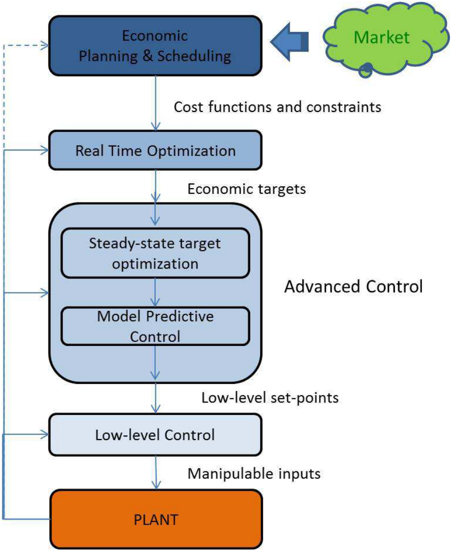

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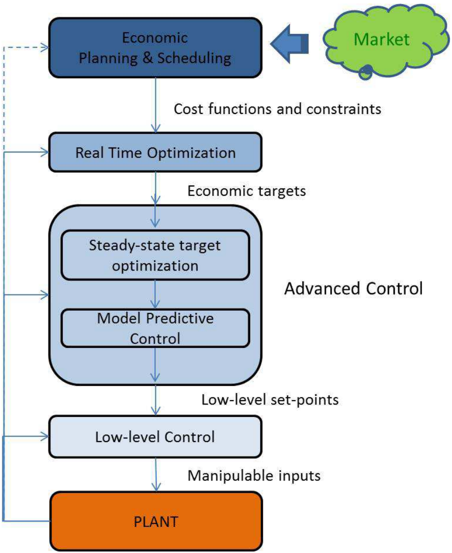

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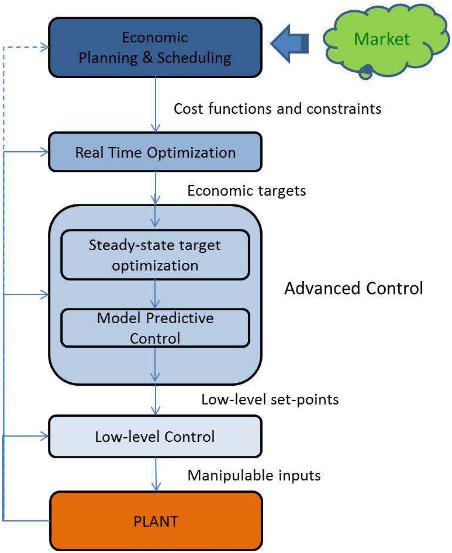

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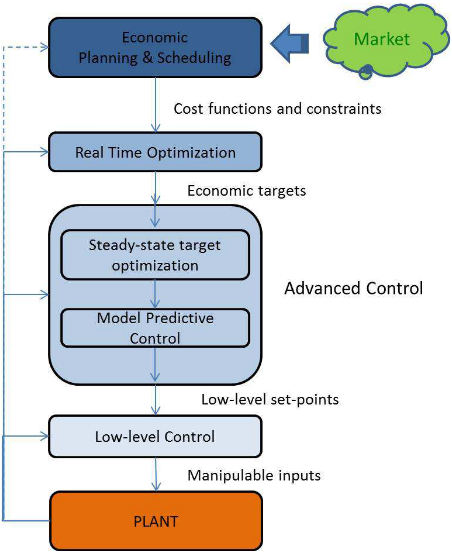

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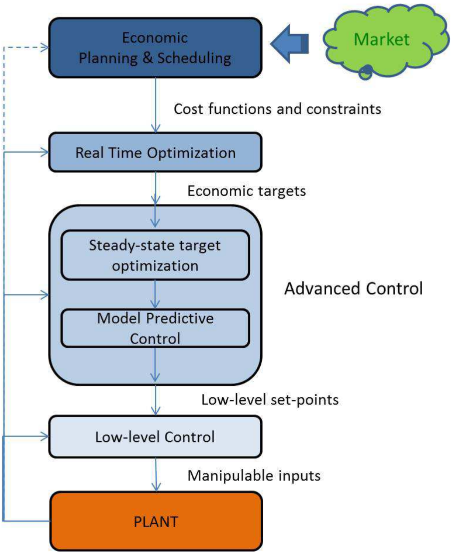

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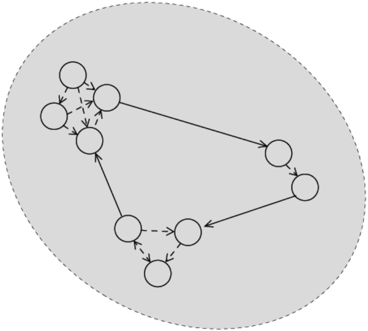
Different interconnected units

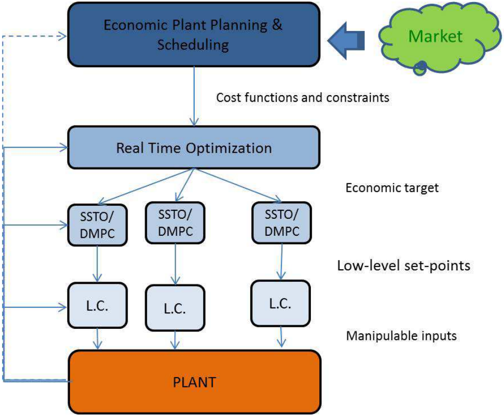


1. Centralized control: high computational burden.
2. Decentralize control: interactions between the different subsystems not considered.
3. Distributed control: "*agents share open-loop information in order to improve closed-loop performance*". (Rawlings & Mayne, 2009).

Different distributed control strategies

1. Noncooperative control: agents' objectives compete (Camponogara et al., 2002; Dunbar, 2007).
   - Nash equilibrium: neither player can improve his position given the other players' decision.
2. Dual decomposition: Lagrange multipliers as prices in a market mechanism, to achieve an agreement (Rantzer, 2009; Negenborn et al., 2009).
3. Cooperative control: agents share a common objective, which can be considered as the overall plant objective (Rawlings & Mayne, 2009; Stewart et al., 2010).
   - Negotiation: agents make different proposals, one of them is chosen based on some criterion (Maestre et al., 2011).

Cooperative MPC para tracking (Ferramosca et al., 2013)



1. SSTO + MPC in one optimization loop.
2. Distributed dynamic optimization.
3. Centralized target problem.
4. Convergence to centralized optimum.

Problem statement

- Consider a system described by a discrete time invariant linear system:

$$\begin{aligned} x^{+} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

- $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, x^+ is the successor state.
- Hard constraints on state and control input:

$$(x(k), u(k)) \in \mathcal{Z} = \mathcal{X} \times \mathcal{U}$$

- $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{U} \subset \mathbb{R}^m$ compact convex polyhedra containing the origin in their interior.

Assumption

The pair (A,B) is stabilizable and the state is measured at each sampling time.

Problem statement (2)

- Steady state: (x_s, u_s, y_s) such that $x_s = Ax_s + Bu_s$, and $y_s = Cx_s + Du_s$ and

$$\begin{bmatrix} A - I_n & B & \mathbf{0}_{p,1} \\ C & D & -I_p \end{bmatrix} \begin{bmatrix} x_s \\ u_s \\ y_s \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{n,1} \\ \mathbf{0}_{p,1} \end{bmatrix} \tag{2}$$

- This parametrization is possible if and only if $\mathit{rank} = n + p$. Otherwise, use a linear combination of a vector $\theta \in \mathbb{R}^m$ (Limon et al., 2008).
- Sets of admissible equilibrium states, inputs and outputs:

$$\mathcal{Z}_s = \{(x, u) \in \mathcal{X} \times \mathcal{U} \mid x = Ax + Bu\} \tag{3}$$

$$\mathcal{X}_s = \{x \in \mathcal{X} \mid \exists u \in \mathcal{U} \text{ such that } (x, u) \in \mathcal{Z}_s\} \tag{4}$$

$$\mathcal{Y}_s = \{y = C_x + D_u \mid (x, u) \in \mathcal{Z}_s\} \tag{5}$$

Distributed partition of the plant

- System (1) can be partitioned in M subsystems of the form:

$$
\begin{aligned}
x_i^+ &= A_i x_i + \sum_{j=1}^{M} \bar{B}_{ij} u_j \\
y_i &= C_i x_i + \sum_{j=1}^{M} \bar{D}_{ij} u_j
\end{aligned} \tag{6}
$$

where $x_i \in \mathbb{R}^{n_i}$, $u_j \in \mathbb{R}^{m_j}$, $y_i \in \mathbb{R}_i^p$, $A_i \in \mathbb{R}^{n_i \times n_i}$, $\bar{B}_{ij} \in \mathbb{R}^{n_i \times m_j}$, $C_i \in \mathbb{R}^{p_i \times n_i}$ and $\bar{D}_{ij} \in \mathbb{R}^{p_i \times m_j}$.

- Without loss of generality, it is considered that

$$u = (u_1, \cdots, u_M)$$

- For the two players game:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^+ = \begin{bmatrix} A_1 & \\ & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \bar{B}_{11} \\ \bar{B}_{21} \end{bmatrix} u_1 + \begin{bmatrix} \bar{B}_{12} \\ \bar{B}_{22} \end{bmatrix} u_2$$

Cooperative MPC for tracking

Players share a common objective: overall plant cost function.

- Cooperative MPC (Stewart et al., 2010):

$$V_N^c(x, y_t; \mathbf{u}) = \sum_{j=0}^{N-1} \|x(j) - x_t\|_Q^2 + \|u(j) - u_t\|_R^2 + \|x(N) - x_t\|_P^2$$

- Cooperative MPC for tracking (Ferramosca et al., 2013):

$$V_t(x, y_t; \mathbf{u}, \hat{y}_s) = \sum_{j=0}^{N-1} \|x(j) - \hat{x}_s\|_Q^2 + \|u(j) - \hat{u}_s\|_R^2 + \|x(N) - \hat{x}_s\|_P^2 + V_O(\hat{y}_s, y_t)$$

Features:

- Artificial steady state and input as decision variables.
- Penalizing the deviation of the predicted trajectory with the artificial steady conditions.
- Centralized offset-cost function.
- The system is stirred to *any* equilibrium point in N steps.

Optimization problem (2 players game)

Each i-th agent solves an iterative decentralized optimization problem

$$
\begin{aligned}
(\mathbf{u}_i^*, \hat{y}_{s,i}^*) \quad &= \quad \arg\min_{\mathbf{u}_i, \hat{y}_{s,i}} V_t(x, y_t; \mathbf{u}, \hat{y}_s) \\
s.t. \quad & x_q(j+1) = A_q x_q(j) + \sum_{\ell=1}^{2} B_{q\ell} u_\ell(j), \; q \in \mathbb{I}_{1,2} \\
& x(0) = (x_1, x_2), \\
& \mathbf{u}_\ell = \mathbf{u}_\ell^{[p]}, \quad \ell \in \mathbb{I}_{1,2} \setminus i, \\
& x(j) \in \mathcal{X}, \\
& (u_1(j), u_2(j)) \in \mathcal{U}, \quad j = 0, ..., N-1 \\
& (x(N), \hat{y}_s) \in \Omega_{t,K}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{u}_1^{[p+1]} &= w_1 \mathbf{u}_1^* + w_2 \mathbf{u}_1^{[p]} \\
\mathbf{u}_2^{[p+1]} &= w_1 \mathbf{u}_2^{[p]} + w_2 \mathbf{u}_2^* \\
\hat{y}_s^{[p+1]} &= w_1 \hat{y}_{s,1}^*(x, \mathbf{u}_2^{[p]}) \\
&\quad + w_2 \hat{y}_{s,2}^*(x, \mathbf{u}_1^{[p]}) \\
w_1 + w_2 &= 1 \quad w_1, w_2 > 0
\end{aligned}
$$

Stop:

- $\mathbf{u}_1^{[p+1]} = \mathbf{u}_1^{[p]}$
- $p = \bar{p}$

- Centralized invariant set for tracking (Limon et al., 2008; Ferramosca et al., 2009).
- Jacobi (simultaneous) solution (Bertsekas & Tsitsiklis, 1997, pp. 219-223) and convex update.
- Iterate $(\mathbf{u}_1^{[0]}, \mathbf{u}_2^{[0]})$: *warm-start* algorithm.

The *warm start* algorithm (Ferramosca et al., 2013)

1. First candidate:

$$\tilde{\mathbf{u}}(k+1) = \{u(1;k), \ldots, u(N-1;k), u_c(N)\}$$

where

$$u_c(N) = Kx(N) + L\hat{y}_s^*(k)$$

centr. terminal control law.

2. Second candidate:

$$\hat{\mathbf{u}}(k+1) = \{\hat{u}_c(0), \ldots, \hat{u}_c(N-1)\}$$

where

$$\begin{aligned} \hat{x}(0) &= x(k+1) \\ \hat{x}(j+1) &= (A+BK)\hat{x}(j) + BL\hat{y}_s^*(k), \; j \in \mathbb{I}_{1:N-2} \\ \hat{u}_c(j) &= K\hat{x}(j) + L\hat{y}_s^*(k) \end{aligned}$$

3. **IF** $(x(k+1), \hat{y}_s^*(k)) \in \Omega_{t,K}$ **AND** $V_N(x(k+1), y_t, \hat{\mathbf{u}}) \leq V_N(x(k+1), y_t, \tilde{\mathbf{u}})$
   SET
   $$\mathbf{u}(k+1)^{[0]} = (\hat{\mathbf{u}}(k+1), \hat{y}_s^*(k))$$
   ELSE
   $$\mathbf{u}(k+1)^{[0]} = (\tilde{\mathbf{u}}(k+1), \hat{y}_s^*(k))$$

Remark

- *When $(x(k+1), \hat{y}_s^*(k)) \in \Omega_{t,K}$, the distr. MPC should work better than the centralized terminal controller.*
- *If not, i.e. $V_N(x(k+1), \hat{\mathbf{u}}) \leq V_N(x(x+1), \tilde{\mathbf{u}})$, chose the centralized terminal control law as warm start.*

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Properties

Stability: for any y_t, closed-loop A-stable and converges to

$$y_s = \arg \min_{\hat{y}_s \in \lambda \mathcal{Y}_s} V_O(\hat{y}_s, y_t)$$

- Convexity.
- Recursive feasibility: warm start and centralized terminal constraint.
- Sub-optimal MPC (Scokaert et al., 1999).

Optimality: for any y_t, centralized optimum.

- For coupled and uncoupled constraints. For any $\bar{p} \geq 1$.
- Centralized offset cost function (SSTO).
- Warm start.

Larger domain of attraction: set of initial conditions admissibly steerable to *any* steady state.

- Constraints do not depend on y_t.

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The 4 tanks plant[1]

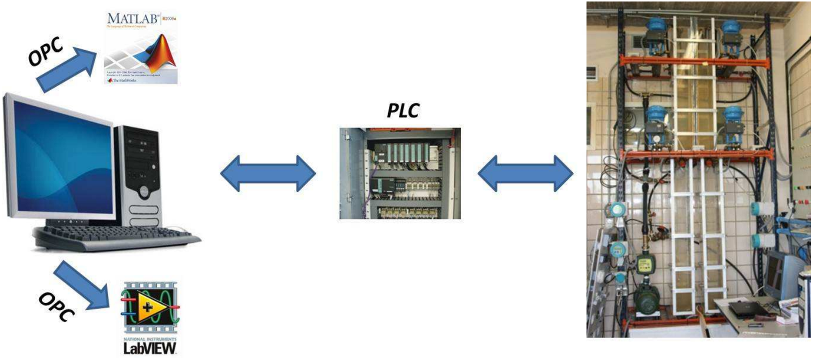
Interesting features:

1. All the states are measurable.
2. The outputs are strongly coupled.
3. The system is nonlinear.
4. The states and inputs are constrained.

$$
\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_a}{A_1}q_a \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_b}{A_2}q_b \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_b)}{A_3}q_b \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_a)}{A_4}q_a
\end{aligned}
$$

[1] Detailed description of the plant and simulation model at http://www.ict-hd-mpc.eu.

The 4 tanks plant: setup at Univ. de Sevilla



- 4 pressure sensors: Siemens Sitrans P 7MF4020.
- 4 electromagnetic flowmeters Siemens.
- 4 pneumatic control valves: Siemens VC 101.
- Pumping system: Pump SACI K41T 4HP.
- Maximum Levels detected by floats switches.
- PLC: Siemens S7-200.

The 4 tanks plant: *LabView*©

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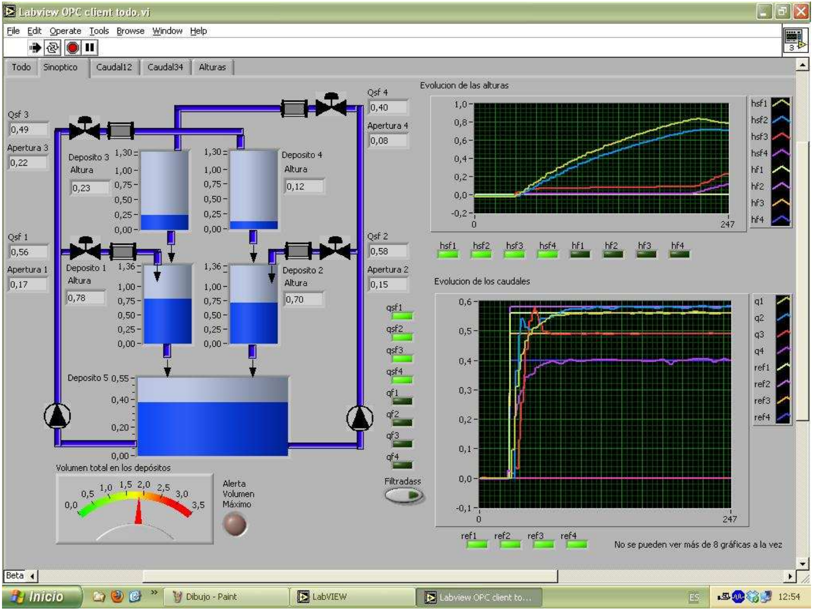
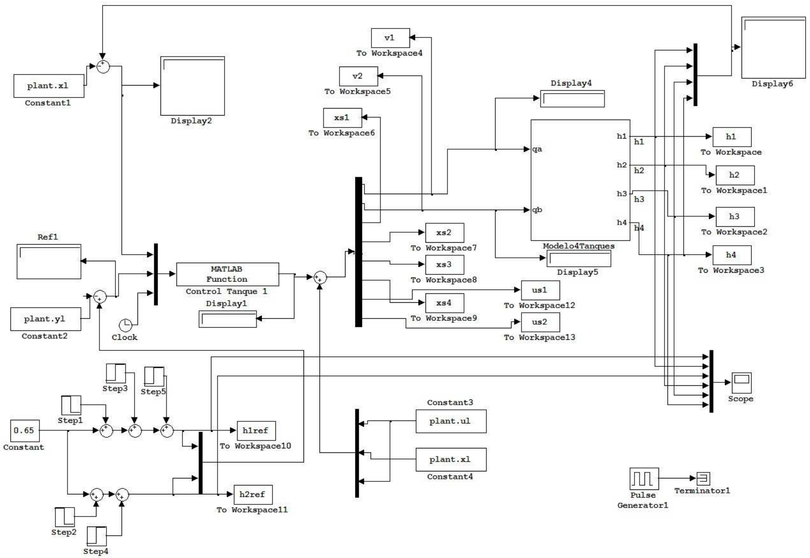
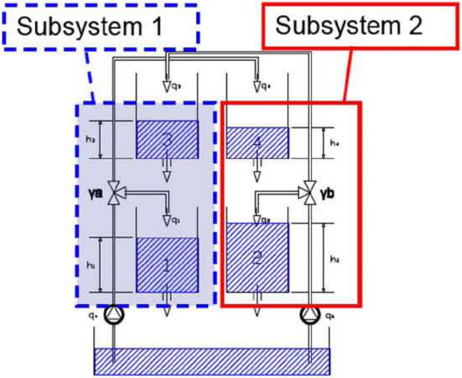


The 4 tanks plant: *Simulink*©



The distributed model



Linearization:

- $\bar{u}_o = (1.63, 2)$.
- $\bar{x}_o$ as the levels s. t. $(x - \bar{x}_o) = A(x - \bar{x}_o) + B(u - \bar{u}_o)$.

Distributed model:

$$\frac{dx_1}{dt} = \begin{bmatrix} \frac{-1}{\tau_1} & \frac{A_3}{A_1\tau_3} \\ 0 & \frac{-1}{\tau_3} \end{bmatrix} x_1 + \begin{bmatrix} \frac{\gamma_a}{A_1} \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ \frac{(1-\gamma_b)}{A_3} \end{bmatrix} u_2$$

$$\frac{dx_2}{dt} = \begin{bmatrix} \frac{-1}{\tau_2} & \frac{A_4}{A_2\tau_4} \\ 0 & \frac{-1}{\tau_4} \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ \frac{(1-\gamma_a)}{A_4} \end{bmatrix} u_1 + \begin{bmatrix} \frac{\gamma_b}{A_2} \\ 0 \end{bmatrix} u_2$$

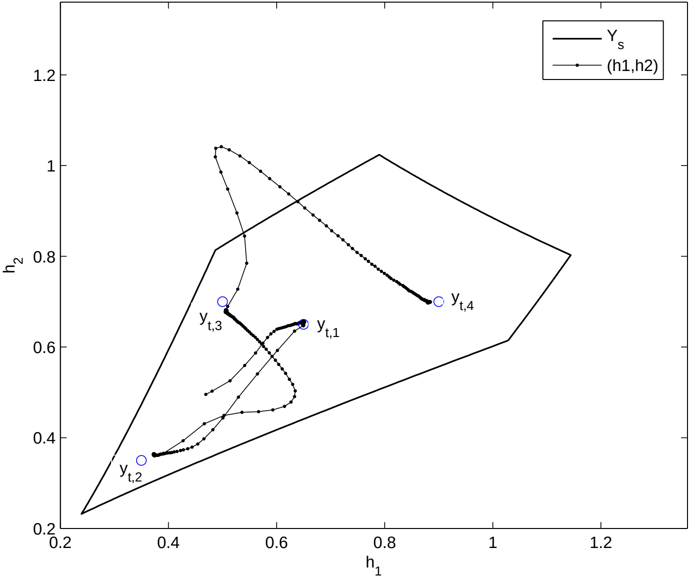
where $x_1 = (h_1, h_3)$, $x_2 = (h_2, h_4)$, $u_1 = q_a$ and $u_2 = q_b$.

Experiment

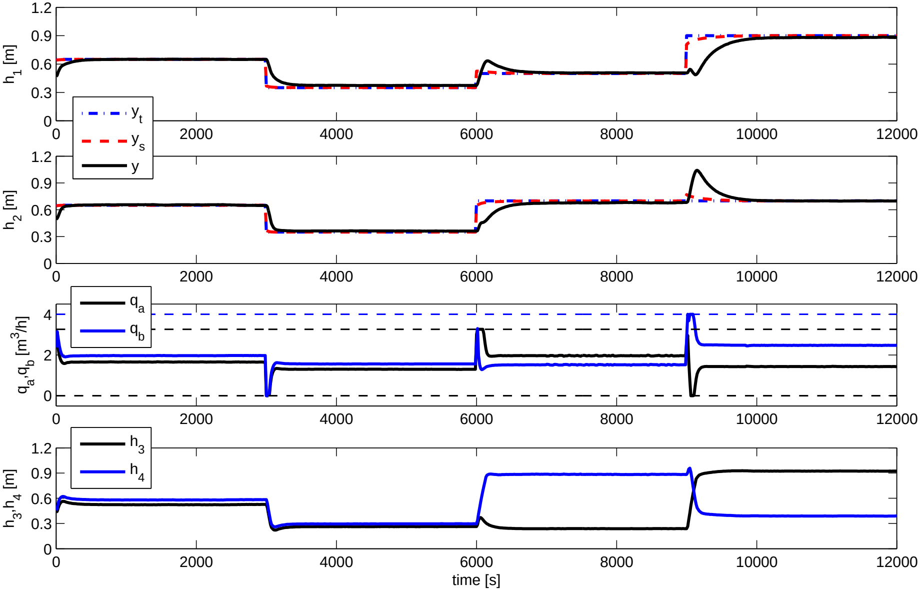
- Initial condition: $x_0 = (0.47, 0.49, 0.44, 0.46)$.
- 4 changes of reference: $y_{t,1} = (0.65, 0.65)$, $y_{t,2} = (0.35, 0.35)$, $y_{t,3} = (0.50, 0.70)$ and $y_{t,4} = (0.90, 0.70)$.
- MPC setup: N=5, $Q = I_4$, $R = 0.01 I_2$, $w_{1,2} = 0.5$.
- $K = K_{LQR}$. P: solution of the Riccati equation.
- $V_O(y_s, y_t) = \|y_s - y_t\|_T^2$ where $T = 100 I_2$.
- $\bar{p} = 1$.
- Constraints:

	Value	Unit	Description
H_{1max}	1.36	m	Maximum level of the tank 1
H_{2max}	1.36	m	Maximum level of the tank 2
H_{3max}	1.30	m	Maximum level of the tank 3
H_{4max}	1.30	m	Maximum level of the tank 4
H_{min}	0.2	m	Minimum level in all cases
V_{max}	0.2226	m^3	Maximum water volume
Q_{amax}	3.26	m^3/h	Maximum flow of pump A
Q_{bmax}	4	m^3/h	Maximum flow of pump B
Q_{min}	0	m^3/h	Minimal flow

Experiment: results



Experiment: results (2)



Summary

Cooperative distributed MPC for tracking.

- Players share a common objective.
- Centralized offset cost function and terminal constraint.
- Feasibility ensured for any changing setpoint.
- Convergence to the centralized optimum ensured for any $\bar{p} \geq 1$.
- Domain of attraction larger than standard MPC.

Future works:

- Economic offset cost function: DMPC+RTO, incremental model.
- Distributed terminal constraint (Conte et al., 2012; Rakovic et al., 2010).
- Nonlinear cooperative MPC for tracking.
- Offset free.

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Thanks for your attention!